Solomon Franco on the Zero Point for Trepidation

Bernard R. Goldstein

To Mercè Comes (1949–2010): In memoriam

Key words: Azarquiel, Ibn al-Kammād, Ṣāᶜid al-Andalusī, Thābit Ibn Qurra, Toledan Tables.

Abstract: In the Middle Ages theories of trepidation, or accession and recession, were proposed to account for variable precession of the equinoxes; the values for trepidation were sometimes positive and sometimes negative, depending on the date. Several dates when the value was 0º have been noted in the literature, and the Hebrew text of Solomon Franco (14th century) confirms two of them: about 18 years before the Hijra, and about 40 years before the Hijra.

The Spanish astronomer, Solomon Franco (14th century), compiled of a set of astronomical tables (or zij) which survives in a unique copy (Vatican, MS Heb. 498), identified by Y. Tzvi Langermann.[1] This zij is a Hebrew version of *al-Zīj al-Muqtabas* by Ibn al-Kammād (*fl*. 1115), extant in a Latin version

[1] Langermann 1984; see also Goldstein 2009, p. 163. For a detailed description of Vatican, MS Heb. 498, see Richler 2008, p. 428. Richler calls attention to various dates in the second half of the 14th century that appear in the text.

(Madrid, Biblioteca Nacional de España, MS 10023) but not in the original Arabic.[2]

Theories of trepidation (or accession and recession) were proposed in the Middle Ages to account for variable precession of the equinoxes, that is, it was believed by many medieval astronomers that precession, the motion in longitude of the fixed stars with respect to the vernal equinox, was not constant.[3] In constructing tables for theories of trepidation in which the values were sometimes positive and sometimes negative, a zero point was sought, that is, the moment in time when the value for trepidation was 0º.[4] This moment was described by different authors and they did not agree with each other. In a discussion of this issue by Julio Samsó, two dates were given: 15 Jan. 581 according to Ibn ᶜAzzūz (14th century), and 24 Jan. 581 according to Ibn al-Kammād (15155 days and 15146 days before the Hijra, respectively, or about 41 years).[5]

Mercè Comes reported that the value for trepidation was zero 50 years before the Hijra according to the Toledan astronomers, whereas for

[2] Langermann (1993, pp. 31, 41) noted that Solomon Franco is the author of a supercommentary on Abraham Ibn Ezra's commentary on the Pentateuch, in which he offered astrological interpretations of biblical passages; see also Schwartz 2005, pp. 167–178. In Vatican, MS Heb. 498, 2a, Franco refers to the astrological treatises of Abraham Ibn Ezra, and in another Hebrew text Franco says that he was a student of the 14th-century Spanish astronomer, Joseph Ibn Waqār (Oxford, Bodleian Library, MS Hunt. 559, 90b:12–19). On the zij of Joseph Ibn Waqār preserved in Munich MS Heb. 230, see Castells 1996, and Chabás and Goldstein 1997, pp. 95–96. On f. 4a of the Munich manuscript Ibn Waqār refers to the zij of Ibn al-Kammād. The geographical coordinates of Córdoba are given in Vatican, MS Heb. 498, f. 8b, but they may be due to Ibn al-Kammād who worked in that city, rather than to Franco. Note, for example, that the table for the oblique ascensions for Córdoba on ff. 55b-56a is the same as that in the Latin version of Ibn al-Kammād's zij, which strongly suggests that the data for this city comes from Ibn al-Kammād. For an analysis of Ibn al-Kammād's zij, see Chabás and Goldstein 1994.

[3] This motion was usually ascribed to a sidereally fixed point, Aries 0º. On the theories of trepidation see, e.g., Rajep 1996; Mercier 1996; Comes 1996.

[4] For example, for Pseudo-Thābit trepidation ranged from –10;45º to +10;45º (see n. 8, below), whereas for Ibn al-Kammād it ranged from –9;59º to +9;59º (Madrid, MS 10023, 35v). In Vatican, MS Heb. 498, 40a, the entries in the table for trepidation (with the heading, "table of the equation of access of the head of Aries") are the same as in the Latin version of Ibn al-Kammād's table (with the heading, "tabula directionis aduentionis capitis arietis"), transcribed in Chabás and Goldstein 1994, p. 25.

[5] See Samsó 1997, p. 108. After a careful analysis of the parameters in Azarquiel's tables for trepidation, arranged separately in three calendars (Julian, Arabic, and Persian), Mielgo (1996, p. 178) argued that the zero point corresponds to a date between 1 Jan. and 20 Mar. 581.

Azarquiel (d. 1100) it took place 40 years before the Hijra.[6] However, in the Toledan Tables, the zero point was in year 604 (i.e., 18 years before the Hijra).[7] These dates confirm the information given by Solomon Franco in the canons to his tables (for the text and translation of the relevant passage, see below). In Pseudo-Thābit's table for trepidation, the radix is 1;34,2º and the motion in 1 Arabic year is 0;5,9º;[8] hence, dividing 1;34,2º by 0;5,9º, we get about 18y and 92d, or about 18 Arabic years, 3m (where m = 29;30d), 5d, which is close to the value cited in Franco's canons in the name of Ṣāᶜid al-Andalusī.

This brief passage by Solomon Franco is of considerable value, for it adds confidence to what we know indirectly about the zijes of Ibn al-Kammād and Ṣāᶜid al-Andalusī, which are not extant in the original Arabic.

Hebrew text

[9ב: 5-] השער השמיני. אמר המחבר דע כי קודם מנין הגר בי״ח שנה וארבעה חדשים וז׳ ימים היה התנועה לא מוקדמת ולא מתאחרת כמו שאמר אבן צעד בלוחותיו. ובלוחות אלמקתבץ היה קודם פרט הגר מ״ב שנה וז׳ חדשים וי״ז יום. ובשלמות תתקנ״ה שנה וז׳ חדשים וי״ג ימים למנין פרט הגר יתחיל התנועה להתאחר.

Translation

[Vatican, MS Heb. 498, 9b:–5] Chapter 8. The author said: Know that 18 years, 4 months, and 7 days before the reckoning (*minyan*) of Hagar there was no motion of access or recess, as Ibn Ṣaᶜad [read: Ibn Ṣāᶜid] recorded in his zij. And in the *Muqtabaṣ* [read: *Muqtabas*] zij this [moment] occurred 42 years and 7 months and 17 days prior to the era (*peraṭ*) of Hagar. And when 955 years and 7 months and 13 days are completed according to reckoning of the era (*minyan peraṭ*) of Hagar, the motion of recess will begin.

[6] Comes 2001, p. 331. See also Millás 1943–1950, p. 338. For a translation of Azarquiel's text, see Samsó 1994, p. 22: Samsó computes 41 years rather than about 40 years which appears in the text.
[7] Mercier 1996, pp. 306–307.
[8] Neugebauer 1962, p. 296. For the corresponding entries in the Toledan Tables, see Pedersen 2002, p. 1545.

Comments

1. Ibn Ṣaᶜad is probably a scribal error for Ibn Ṣāᶜid, that is, Ṣāᶜid al-Andalusī (11th century), who was the head of a school of astronomers in Toledo.[9]

2. Hagar (the mother of Ishmael in the Bible) is used here to refer to Muslims in general. The era (or the epoch) of Hagar refers to the Hijra.

3. It seems that according to this author, after about 998 Arabic years (42½ + 955½) of access, recess will begin — this is the time from 0º to 90º of the argument (or a quarter of the period).[10] There are distinct periods for each of Azarquiel's three models for trepidation; we will see that his third model has the parameter we seek. The value in the first model is 3792 Julian years.[11] Dividing this value by 4, we get 948 Julian years, which corresponds to about 977 Arabic years in the Hijra calendar; this value is too small compared to the value in Franco's text. The value in the second model is 5150 Julian years,[12] which is too big compared to the value in Franco's text. The value in the third model is 3874 Julian years, which is approximately 3993 Arabic years.[13] Dividing 3993 Arabic years by 4, we get about 998 Arabic years, in agreement with Franco's text.[14]

In the table corresponding to Azarquiel's third model, the radix for the Hijra is 3;51,11º, and the mean motion in 1 Arabic year is 0;5,24,32º.[15] I then compute the interval from the moment when the value for trepidation was zero to the time of the Hijra to be about 42y 263d (= 3;51,11º / 0;5,24,32º), or about 42y 8m (of 29;30d), 7d, which is reasonably close to the value cited by Franco (about 42½ years). To get exact agreement for the time before the Hijra, the motion per Arabic year would have to be 0;5,25,27º.

[9] See Richter-Bernburg 1987.
[10] Cf. Millás 1943–1950, p. 337. The argument is a linear function of the time since epoch, and the amount of trepidation is a function of the argument such that it reaches its maximum value at 90º of the argument.
[11] See Samsó 1994, p. 15; Millás 1943–1950, p. 316.
[12] See Samsó 1994, p. 21; Millás 1943–1950, p. 318.
[13] See Samsó 1994, pp. 22, 29; Millás 1943–1950, p. 318.
[14] Mancha (1998) argued on other grounds that Ibn al-Kammād used Azarquiel's third model.
[15] See Chabás and Goldstein 1994, pp. 24, 30; Millás 1943–1950, p. 324.

4. The Hebrew term, *peraṭ*, usually means 'specific', 'detail', or 'particular', but at the beginning of chapter 8 it seems to mean 'era', as it does earlier in the text where one finds *peraṭ edom* for the Christian calendar (instead of the usual Hebrew expression for the 'era of the incarnation'),[16] and *peraṭ adam* for the Jewish calendar (rather than the usual 'era of creation').[17] However, on f. 61a of the same manuscript (in the heading of one of the tables appended to the canons, corresponding to the table in the Latin version in Madrid, MS 10023, 54v), the excess of revolution in hours and minutes (over a year of 365 days) is called *yitron* [excess of] *ha-peraṭim*. The meaning here of *peraṭ* does not seem to conform to its meaning in the passage cited above in chapter 8.

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[16] In medieval Hebrew texts Christian Spain was identified as the land of Edom: see, e.g., Cohen 1967, pp. 62 and 96.

[17] The Arabic expression corresponding to *peraṭ adam* is *ta'rīkh ādam*: see, e.g., Sachau [1878] 1923, p. 144 (Arabic text).

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