Bounded orbits

Ludomir Newelski

Instytut Matematyczny
Uniwersytetu Wrocławskiego

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Set-up

T is a countable complete theory
M is a model of T
G is a group definable in M
for simplicity: $G = M$.
G acts on $S(M)$, by left translation:
for $g \in G, p \in S(M)$

$$g \cdot p = \{\varphi(g^{-1}x) : \varphi(x) \in p\}$$

We work in a monster model $\mathfrak{C}$
So:
$G^{\mathfrak{C}}$ acts on $S(\mathfrak{C})$.
Often we skip $\mathfrak{C}$ in $G^{\mathfrak{C}}$.

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Motivating conjecture

A conjecture of Marcin Petrykowski

If there is a bounded G-orbit in $S(\mathfrak{C})$, then G is definably amenable.

Explanation

Definably amenable means there is a left-invariant measure on the family of definable subsets of G (a left-invariant Keisler measure).
Bounded means of cardinality much smaller than $\|\mathfrak{C}\|$.
Is it really an explanation ?
What does bounded exactly mean ?

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Assume $p \in S(\mathfrak{C})$ and Gp has bounded size.

Let $Gp = \{p_\alpha : \alpha < \kappa\}$, $p = p_0$.

Choose a small $M \prec \mathfrak{C}$ such that all the types $p_\alpha \restriction_M, \alpha < \kappa$ are distinct.

Let $q_\alpha = p_\alpha \restriction_M \in S(M)$, $\alpha < \kappa$.

So every q_α extends uniquely to a type in the orbit Gp.

We may also assume $G^M \cdot q_0 = \{q_\alpha : \alpha < \kappa\}$.

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Upside down

Now assume $q = q_0 \in S(M)$ and $G^M q = \{q_\alpha : \alpha < \kappa\}$

Question

Does there exist $p \in S(\mathfrak{C})$ extending q_0 such that every type q_α extends uniquely to a type in Gp and also every type in Gp extends some q_α ?

(In particular, such a Gp would be a bounded orbit...) Call a type p as above good.

Bad type

Call a partial type $r(x) = r(x, \bar{a})$ of size κ, consistent with q_0, bad if for some $g \in G$ the set

$$gr \wedge \bigvee_{0<\alpha<\kappa} [q_\alpha]$$

is contradictory and also the set $gr \wedge r$ is contradictory.

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Whether a given type $r(x, \bar{a})$ is bad, depends only on $tp(\bar{a}/M)$.

A type $p \in S(\mathfrak{C})$ containing q_0 is good iff p contains no bad type.

Hence:

A good type exists iff in $S(\mathfrak{C})$

$$(*) \bigcap_{\text{bad } r} \bigcup_{\varphi \in r} [\neg\varphi] \neq \emptyset$$

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Absoluteness issue

Given q and M as above, we can ask if there is a bounded orbit in $S(\mathfrak{C})$ related to q as in the question.

Does the answer not depend on $\mathfrak{C}$?

Assume $\mathfrak{C}' \succ \mathfrak{C}$ and $(*)$ holds in $\mathfrak{C}$. Does $(*)$ hold in $\mathfrak{C}'$?

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A (simplified) generalized set-up

Assume $\Phi = \{\varphi_n(x, y) : n < \omega\}$ and $s(y)$ is a type over $\emptyset$.
For $A \subseteq \mathfrak{C}$ let

$$X(A) = \bigcap_{a \in s(A)} \bigcup_{n<\omega} [\varphi_n(x, a)] \subseteq S(\mathfrak{C})$$

In fact, $\subseteq S(A)$.

Questions

1. Suppose $X(\mathfrak{C}) \neq \emptyset$ and $\mathfrak{C}' \succ \mathfrak{C}$. Is $X(\mathfrak{C}') \neq \emptyset$?
2. Suppose $X(\mathfrak{C}) = \emptyset$. What is the minimal $\kappa = \kappa(\Phi)$ such that for some $A \subseteq \mathfrak{C}$ of power κ, $X(A) = \emptyset$?

Let $\mu = sup\{\kappa(\Phi) : \Phi, T \text{ countable}\}$.
What is μ ?
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A partial result on the motivating conjecture

Theorem (M.Petrykowski)

If for some $p \in S(\mathfrak{C})$, the orbit Gp is bounded, then G^∞ exists.

Explanation

G^∞_A is the smallest A-invariant subgroup of G, of bounded index. If for every A, $G^\infty_A = G^\infty_\emptyset$, we call this group the ∞-component of G, or G^∞.

Absoluteness of existence of G^∞

1. If for some A, we have that $G^\infty_A \neq G^\infty_\emptyset$, then this holds for some countable A.
2. Existence of G^∞ is absolute both ways:
(a) it does not depend on the monster model,
(b) it does not depend on the set-theoretical universe.

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(a) it does not depend on the monster model,
(b) it does not depend on the set-theoretical universe.

A local version

In the theorem, a vague assumption of existence of a bounded orbit implies an absolute conclusion:
existence of G^∞.

Theorem (A local, absolute version)

Assume M is κ^+-saturated, $p \in S(M)$ and $|Gp| < 2^\kappa$. Then G^∞ exists.

Another Hanff number

Assume the assumption of the theorem holds for some κ (that causes G^∞ exist).
What is the minimal such κ then ?
If G^∞ exists by this reason, how far do we have to seek for the relevant κ ?

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Some topological dynamics

$S(\mathfrak{C})$ is a $G^{\mathfrak{C}}$-flow, that is, $G^{\mathfrak{C}}$ acts on $S(\mathfrak{C})$ by homeomorphisms.

Definitions

1. A type $p \in S(\mathfrak{C})$ is almost periodic if the sub-flow $\mathrm{cl}(Gp)$ is minimal.
2. $APer = \{p \in S(\mathfrak{C}) : p \text{ is almost periodic}\}$.
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Bounded minimal flow

Assume for some $p \in S(\mathfrak{C})$, the orbit Gp is bounded. Then for some almost periodic type $q \in S(\mathfrak{C})$, the minimal flow $\mathrm{cl}(Gq)$ is bounded.

Proof.

Since Gp is bounded, also $\mathrm{cl}(Gp)$ is bounded.

$$|\mathrm{cl}(Gp)| \leq 2^{2^{|Gp|}}$$

But $\mathrm{cl}(Gp)$ is a sub-flow, hence it contains a minimal sub-flow, that is bounded, too. □

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Bounded *WGen*

Hence, if there is a bounded orbit in $S(\mathfrak{C})$, then there is a bounded orbit consisting of weakly generic types. Now consider the case, where *WGen* is bounded.

Definition

Let $\varphi(x, y)$ be a formula.Define an equivalence relation $\sim_\varphi$:

$$a \sim_\varphi b \iff \varphi(x, a) \triangle \varphi(x, b) \text{ is not weakly generic}$$

Since *WGen* is bounded, $\sim_\varphi$ is a bounded invariant equivalence relation, with $\leq 2^{\aleph_0}$ classes.

Absolute bound on *WGen*

1.Assume *WGen* is bounded.Then $|WGen| \leq 2^{2^{\aleph_0}}$, and this does not depend on the monster model, i.e. it is absolute model-theoretically..
2. The boundedness of *WGen* is absolute set-theoretically, too.

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Example

There is a (semi)-example, where *WGen* is bounded, of size $2^{2^{\aleph_0}}$.

Definition

Let $p \in WGen$. We say that p is countably stationary if for some countable $A \subset \mathfrak{C}$, p is the only weakly generic type extending $p \upharpoonright_A$.

Lemma

Assume $|WGen| \leq 2^{\aleph_0}$ and $2^{\aleph_0} < 2^{\aleph_1}$. Then there is a type $p \in WGen$, that is countably stationary.

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If not, build a tree of weakly generic types of height $\aleph_1$, getting $2^{\aleph_1}$-many of types in *WGen*. □

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Example

Assume T has NIP and G has fsg. Then *WGen* consists of generic types and is absolutely bounded by $2^{\aleph_0}$.

Look into the papers on NIP and groups by Hrushovski, Pillay, Peterzil [HPP].

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Assume WGen is absolutely bounded by $2^{\aleph_0}$. Then there is a countably stationary type in WGen.

The case of absolutely very bounded *WGen*

Definition

We say that *WGen* is absolutely bounded by $2^{\aleph_0}$ if this bound persists in any forcing extension of the set-theoretical universe underlying our considerations.

Example

Assume T has NIP and G has fsg. Then *WGen* consists of generic types and is absolutely bounded by $2^{\aleph_0}$.

Look into the papers on NIP and groups by Hrushovski, Pillay, Peterzil [HPP].

Theorem

Assume WGen is absolutely bounded by $2^{\aleph_0}$. Then there is a countably stationary type in WGen.

Proof

The conclusion of the theorem says that

There is a countable weakly generic type $p = \{\varphi_n(x, a_n) : n < \omega\}$ that extends uniquely to a type in *WGen*.

The type p is determined by the tuple $\bar{a} = \langle a_n \rangle_{n<\omega}$ of the parameters and the function $f : \omega \rightarrow L$, mapping n to φ_n.. Also just the type $q(\bar{y}) = \mathrm{tp}(\bar{a}) \in S_\omega(\emptyset)$ matters. So the conclusion says:

$(*)(\exists q(\bar{y}), f)($ the type p determined by q and f is weakly generic and for every formula $\psi(x, b)$, at most one of $p \cup \{\psi(x, b)\}$, $p \cup \{\neg\psi(x, b)\}$ is weakly generic).

The fact, that $\varphi(x, a)$ is weak generic is a Borel property of $\mathrm{tp}(a)$ (more exactly: F_σ), hence $(*)$ is a Σ^1_2-sentence of a Polish space.

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By Shoenfield absoluteness lemma, $(*)$ is absolute between various models of ZFC.

In our situation we can extend the set-theoretical universe V (by means of forcing) to a universe V', where $2^{\aleph_0} < 2^{\aleph_1}$ holds.

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Assume T has *NIP* and G has *fsg*. Then there is a countably stationary weak generic type in *WGen*.

Example

Consider the group S^1 in an o-minimal expansion of the reals. Here every type in *WGen* is generic and countably stationary. But *WGen* is not a Polish space here, so we can not find a common countable set A such that A separates the types in *WGen*.

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Bounded *WGen* and measure

Lemma

Assume WGen is bounded. Then $G^\infty = G^{00} = \mathrm{Stab}(p)$ for any $p \in WGen$.

1. G/G^{00} is a compact topological group (with logic topology), with Haar measure μ, also it is a Polish space.
2. Let $p \in WGen$. There is a bijection $Gp \leftrightarrow G/G^{00}$. Every coset of G^{00} contains exactly one type from Gp.

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Fix a type $p \in WGen$.

Lifting Haar measure to Keisler measure

1.Let $U \subseteq_{\text{def}} \mathfrak{C}$. Let $\varphi(U)$ be the set

$$\{g/G^{00} : U \text{ belongs to the unique type in } Gp \text{ in the coset } g/G^{00}\}$$

2. Let $Mes(\mathfrak{C}) = \{U \subseteq_{\text{def}} (\mathfrak{C}) : \varphi(U) \text{ is measurable}\}$. This is an algebra of sets.
3. For $U \in Mes(\mathfrak{C})$ let $\nu(U) = \mu(\varphi(U))$.
4. ν is a finitely additive left invariant measure on $Mes(\mathfrak{C})$.

Theorem

Assume $p \in WGen$ is countably stationary. Then $Mes(\mathfrak{C})$ consists of all definable sets. In particular, on G there is a left-invariant Keisler measure.

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Using countable stationarity of p one shows that for every $U \subseteq_{def} \mathfrak{C}$, the set $\varphi(U)$ is analytic (that is, Σ^1_1).
Analytic sets are measurable with respect to Haar measure in a Polish group. □

Corollary

Assume WGen is absolutely bounded by $2^{\aleph_0}$. Then G is definably amenable.

This corollary applies in particular to groups with fsg, under *NIP*-assumption.
In this special case Hrushovski and Pillay proved moreover uniqueness of left invariant Keisler measure.
In general we obviously do not have uniqueness.

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Final comments

Example

In the additive group of the reals we have exactly two left-invariant Keisler measures, corresponding to the two weak generic types there.

We proved the conjecture of Petrykowski under a stronger assumption that not only is there a bounded orbit, but that the set $WGen$ is absolutely bounded by $2^{\aleph_0}$. The conjecture is open.

Further research:

- Model-theoretic absoluteness of Ellis semigroup.
- Relations between the subgroups of the Ellis semigroup and the group G/G^{00}.

There are some preliminary results here.

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