Exact inference for the spatial autocorrelation coefficient in linear panel models

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May 29, 2009

Abstract

We analyze the small sample properties of various correlation tests in linear panel models with spatial autoregressive (SAR) or moving average (SMA) errors. The size distortions of some tests can be corrected by applying Monte Carlo (MC) test versions of the statistics leading to exact tests for spatial correlation, while the power remains the same. For the SAR and the SMA specification, we apply the method, proposed by Dufour (1990), to build exact confidence sets for the spatial autocorrelation coefficient by "inverting" spatial (MC) correlation tests. Furthermore, we develop upon our procedure a test based point estimator. Simulation results show that for large time dimensions our estimator do nearly perform as good as the Maximum Likelihood estimator and better for some cases with SMA errors. Besides we deliver exact confidence intervals for our estimates. It shows that Pesaran's (2004) CD test obtains exact inference even if the spatial weights matrix is misspecified.

JEL-Classification: C12, C15, C21, C23

Keywords: Panel data, Spatial autocorrelation, Specification tests, Monte Carlo test, Exact confidence sets

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1 Introduction

Several authors mention substantial computational problems of the Maximum Likelihood (ML) estimator for spatial models if the cross-sectional dimension is large (e.g. Kapoor et al., 2007, Anselin et al., 2008, Kelejian and Prucha, 1999). Kapoor et al. (2007) and Kelejian and Prucha (1999) circumvent this problem for a SAR specification in the panel and cross-sectional case, respectively. They suggest a generalized moments estimator for the spatial autocorrelation coefficient and the variance components of the disturbance process. A feasible generalized least squares (FGLS) procedure obtains an efficient estimation of the regression parameters. However, they do not derive the distributional properties of the autocorrelation parameter.

In this paper, we apply the method, proposed by Dufour (1990), to construct exact confidence sets for the spatial autocorrelation coefficient in linear panel models with spatial autoregressive (SAR) or moving average (SMA) errors (Cliff and Ord, 1973, 1981, Anselin, 1988 and Anselin et al., 2008). These sets are constructed by utilizing tests for correlation among the regression disturbances. To find the best performing test for obtaining exact and as small as possible intervals, we consider small sample properties of various correlation tests under several spatial specifications. The considered tests are Pesaran's (2004) CD test, the statistic proposed by Breusch and Pagan (1980) and the LM^E statistic (Burridge, 1980), extended to the pooled regression model (Anselin et al., 2008). As suggested by Pesaran (2004), we consider also transformations of the CD and the Breusch and Pagan (1980) statistic to test for local, in the sense of spatial correlation. Since most of the tests have distorted sizes when the time dimension is small, we develop a Monte Carlo (MC) test procedure. Results of Monte Carlo simulations show that this procedure ensures the correct size of the tests, while the size adjusted power remains the same.

Furthermore, we develop upon our procedure a test based estimator for the spatial autocorrelation coefficient. The point and the interval estimator are compared in a Monte Carlo simulation to the Maximum Likelihood (ML) estimator (e.g. Anselin, 1988, Elhorst, 2003 and Mur and Angulo, 2007). Our procedure obtains exact confidence intervals (CI), which are in some cases on average smaller than the obtained intervals of the ML estimator. If the spatial weights matrix is misspecified, the CI estimator, based on the CD test, maintains the exact properties. The LM^E test based point estimator does nearly perform as good as the ML estimator and even better for some cases with SMA errors.

The remainder of this paper is as follows: in Section 2 the spatial panel model under the SAR and SMA specification is introduced. Section 3 presents the method of constructing exact confidence intervals for the spatial autocorrelation coefficient and its test based point estimator. The considered tests and the MC test procedure are described in Section 4. In Section 5 the small sample properties of the tests and estimators are discovered, and the latter are compared to the ML estimator. Section 6 concludes.

2 The spatial panel model

The underlying model is the spatial panel error model (Cliff and Ord, 1973, 1981, Anselin, 1988 and Elhorst, 2003). Consider a pooled linear regression model

$$y_t = X_t\boldsymbol{\beta} + e_t, \quad t = 1, ..., T, \tag{1}$$

where y_t is an $N \times 1$ vector of observations of the dependent variable in time t, X_t is an $N \times K$ matrix of observations of explanatory variables, $\boldsymbol{\beta}$ a $K \times 1$ vector of parameters and e_t an $N \times 1$ vector of error terms. The cross-sectional and time dimension is denoted by N and T, respectively. The spatial interdependence is introduced through the error term affecting its covariance structure. We consider two different specifications of the error term process. The SAR specification

$$e_t = \rho W e_t + \epsilon_t, \tag{2}$$

and the SMA specification

$$e_t = \gamma W \epsilon_t + \epsilon_t, \tag{3}$$

where W is a spatial weights matrix of dimension $N \times N$ with zero diagonal and row normalized constants (such that each row sums to unity), ρ and γ are the spatial autocorrelation coefficients with $\rho \in (-1, 1)$ and $\gamma \in (-1, 1)$ (e.g. Mur and Angulo, 2007), and ϵ_t is an $N \times 1$ vector of location specific disturbance terms, with the following properties

$$E[\epsilon_t] = 0, \text{ and } E[\epsilon_t\epsilon_t'] = \sigma_\epsilon^2 I_N. \tag{4}$$

Alternatively, the error term of the SAR model can be expressed as

$$e_t = (I_N - \rho W)^{-1}\epsilon_t. \tag{5}$$

The covariance matrix of e_t is given by

$$E[e_t e_t'] = \sigma_\epsilon^2 \left[I_N - \rho(W + W') + \rho^2 W'W\right]^{-1} = \Omega_{SAR}, \tag{6}$$

which is a non-sparse matrix for $\rho \neq 0$, suggesting that a shock in one location is transmitted to other locations which are not directly connected to each other. Due to the global effect of a shock, the spatial covariance structure in this model is denoted as global (e.g. Anselin et al., 2008 and Fingleton, 2008). For the SMA specification, the error terms become to

$$e_t = (I_N + \gamma W)\epsilon_t, \tag{7}$$

thus the covariance matrix is

$$E[e_t e_t'] = \sigma_\epsilon^2 \left[I_N + \gamma(W + W') + \gamma^2 WW'\right] = \Omega_{SMA}. \tag{8}$$

If W is a first order contiguity matrix, the spatial covariance includes only linkages between first and second order neighbors. Hence, in contrast to the SAR model, the shock-effects are local.

In matrix form the model can be written as

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{e}, \tag{9}$$

where the SAR error process become to

$$\boldsymbol{e} = \left[I_T \otimes (I_N - \rho W)^{-1}\right]\boldsymbol{\epsilon} \tag{10}$$

and the SMA to

$$\boldsymbol{e} = \left[I_T \otimes (I_N + \gamma W)\right]\boldsymbol{\epsilon}, \tag{11}$$

where $\otimes$ denotes the Kronecker product, $\boldsymbol{y} = (y_1', ..., y_T')'$ a $(TN \times 1)$ vector, $\boldsymbol{X} = (X_1', ..., X_T')'$ a $(TN \times K)$ matrix, $\boldsymbol{e} = (e_1', ..., e_T')'$ and $\boldsymbol{\epsilon} = (\epsilon_1', ..., \epsilon_T')'$ are $(TN \times 1)$ vectors.

3 Test based estimation

In this Section, we describe our test based estimation procedure. For ease of illustration, in the following θ denotes the corresponding autocorrelation parameter, ρ (SAR) and γ (SMA).

3.1 Exact confidence sets

To construct exact confidence sets for θ, we proceed from testing the following hypotheses

$$H_0 : \theta = \theta_0 \quad vs. \quad H_1 : \theta \neq \theta_0, \tag{12}$$

for all admissible values of $\theta_0 \in (-1, 1)$. Common tests for spatial correlation face only the case, where $\theta_0 = 0$. To test the more general problem (12) we first specify the panel model under the null hypothesis. The SAR model is specified as

$$\boldsymbol{y} = \boldsymbol{X\beta} + \left[I_T \otimes (I_N - \theta_0 W)^{-1}\right] \boldsymbol{\epsilon}, \tag{13}$$

whereas the SMA as

$$\boldsymbol{y} = \boldsymbol{X\beta} + \left[I_T \otimes (I_N + \theta_0 W)\right] \boldsymbol{\epsilon}. \tag{14}$$

An efficient estimator of this model is the generalized least squares estimator (GLS)

$$\hat{\boldsymbol{\beta}}_0^{GLS} = (\boldsymbol{X}_0^{*\prime}\boldsymbol{X}_0^*)^{-1}\boldsymbol{X}_0^{*\prime}\boldsymbol{y}_0^*, \tag{15}$$

where $\boldsymbol{X}_0^*$ and $\boldsymbol{y}_0^*$ denote the spatially filtered variables under H_0. For the SAR model the variables are filtered by

$$\boldsymbol{X}_0^* = \left[I_T \otimes (I_N - \theta_0 W)\right] \boldsymbol{X} \quad \text{and} \quad \boldsymbol{y}_0^* = \left[I_T \otimes (I_N - \theta_0 W)\right] \boldsymbol{y} \tag{16}$$

and for the SMA by

$$\boldsymbol{X}_0^* = \left[I_T \otimes (I_N + \theta_0 W)^{-1}\right] \boldsymbol{X} \quad \text{and} \quad \boldsymbol{y}_0^* = \left[I_T \otimes (I_N + \theta_0 W)^{-1}\right] \boldsymbol{y}. \tag{17}$$

If the null hypothesis is true, i.e. $\theta = \theta_0$, the spatially filtered estimated residuals

$$\hat{\boldsymbol{\epsilon}}_0 = \boldsymbol{y}_0^* - \boldsymbol{X}_0^* \hat{\boldsymbol{\beta}}_0^{GLS} \tag{18}$$

are spatially uncorrelated. In the other case, when $\theta \neq \theta_0$, some spatial correlation remains, and should be detected by any spatial correlation test. To test for (remaining) spatial correlation various tests for contemporaneous and spatial correlation are considered (Section 4.1). Given any test statistic, $\mathcal{T}$, of a two-sided test for testing $H_0 : \theta = \theta_0$, say $\mathcal{T}(\theta_0)$, the null hypothesis is rejected when the observed value of the statistic, $\widehat{\mathcal{T}}(\theta_0)$, exceeds the upper, c_1, or lower, c_2, critical value, respectively. An exact confidence set for θ with level $1 - \alpha$ is obtained by finding the set of admissible values of θ that are not rejected by the test, e.g.

$$CI^{ts}_{\mathcal{T}}(\alpha) \quad = \quad \left\{\theta_0 \ \in S : c_1 \leq \widehat{\mathcal{T}}(\theta_0) \leq c_2\right\}, \tag{19}$$

where $S = \{\theta : |\theta| < 1\}$. For a one-sided test the exact confidence set becomes to

$$CI^{os}_{\mathcal{T}}(\alpha) \quad = \quad \left\{\theta_0 \ \in S : \widehat{\mathcal{T}}(\theta_0) \leq c_1(\theta_0)\right\}. \tag{20}$$

So that values of θ_0 are assigned to the confidence set, whose corresponding p-values (when testing for $H_0 : \theta = \theta_0$) exceed the nominal level α.

3.2 Test based estimation

Based on this procedure a test based estimator, $\hat{\theta}_{\mathcal{T}}$, for θ is defined as the particular θ_0, where the test statistic, $\mathcal{T}(\theta_0)$, reaches its minimum, i.e.

$$\hat{\theta}_{\mathcal{T}} = \arg\min_{\theta_0}\{\mathcal{T}(\theta_0)\}, \tag{21}$$

such that the remaining spatial correlation is minimized. The regression parameter vector, $\boldsymbol{\beta}$, can be estimated via FGLS

$$\hat{\boldsymbol{\beta}}^{FGLS}_{\mathcal{T}} = (\mathbf{X}^{*\prime}_{\mathcal{T}}\mathbf{X}^{*}_{\mathcal{T}})^{-1}\mathbf{X}^{*\prime}_{\mathcal{T}}\mathbf{y}^{*}_{\mathcal{T}}, \tag{22}$$

where the spatially filtered variables, $\boldsymbol{X}^*_{\mathcal{T}}$ and $\boldsymbol{y}^*_{\mathcal{T}}$, are obtained under SAR as

$$\boldsymbol{X}^*_{\mathcal{T}} = \left[I_T \otimes (I_N - \hat{\theta}_{\mathcal{T}}W)\right]\boldsymbol{X} \quad \text{and} \quad \boldsymbol{y}^*_{\mathcal{T}} = \left[I_T \otimes (I_N - \hat{\theta}_{\mathcal{T}}W)\right]\boldsymbol{y} \tag{23}$$

and under SMA as

$$\boldsymbol{X}^*_{\mathcal{T}} = \left[I_T \otimes (I_N + \hat{\theta}_{\mathcal{T}}W)^{-1}\right]\boldsymbol{X} \quad \text{and} \quad \boldsymbol{y}^*_{\mathcal{T}} = \left[I_T \otimes (I_N + \hat{\theta}_{\mathcal{T}}W)^{-1}\right]\boldsymbol{y}. \tag{24}$$

4 Testing for correlation

4.1 Considered tests

In this Section we present the test statistics used to test, if the filtered residuals (18) are spatially uncorrelated under $H_0 : \theta = \theta_0$.

CD test

The general version of Pesaran's (2004) cross-sectional dependence test is given by

$$CD_0 = \sqrt{\frac{2T}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\varrho}_{0,ij} \right) \quad \overset{\mathrm{d}}{\rightarrow} \quad N(0,1), \tag{25}$$

where

$$\hat{\varrho}_{0,ij} = \frac{\sum_{t=1}^{T} \hat{\epsilon}_{0,it} \hat{\epsilon}_{0,jt}}{\sqrt{\sum_{t=1}^{T} \hat{\epsilon}_{0,it}^2 \sum_{t=1}^{T} \hat{\epsilon}_{0,jt}^2}} \tag{26}$$

is the correlation coefficient of the filtered estimated residuals. The test is valid for N and T tending to infinity. It does not need any spatial weights specification. In the case, where N is relative large compared to T, the power of the CD test can be enhanced by testing for local correlation. Only the correlation among these residuals is considered which are expected to be correlated, because of their contiguity, prespecified by the spatial weights matrix. Following Pesaran (2004), the test statistic (25) can be easily transformed to

$$CD_0^W = \sqrt{\frac{2T}{p}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \mathcal{I}_{(w_{ij}>0)} \; \hat{\varrho}_{0,ij} \right) \quad \overset{\mathrm{d}}{\rightarrow} \quad N(0,1), \tag{27}$$

where p is the number of non-zero elements in W and $\mathcal{I}$ is an indicator function. If W is a non-sparse matrix, this statistic is identical to (25).

BP test

Breusch and Pagan (1980) proposed the following Lagrange Multiplier statistic

$$BP_0 = T \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\varrho}_{0,ij}^2 \quad \overset{\mathrm{d}}{\rightarrow} \quad \chi^2_{N(N-1)/2}. \tag{28}$$

The test is valid for fixed N as T tends to infinity. Similarly to the CD^W test we transform this statistic to test for local correlation as

$$BP_0^W = T \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \mathcal{I}_{(w_{ij}>0)} \hat{\varrho}_{0,ij}^2 \quad \xrightarrow{\text{d}} \quad \chi^2_{p/2}. \tag{29}$$

LM$^{\mathbf{E}}$ test

As another test we use the LM_E statistic (Burridge, 1980), extended to the pooled regression model (e.g. Anselin et al., 2008). It is given by

$$LM_0^E = \frac{\left[\hat{\boldsymbol{\epsilon}}_0'(I_T \otimes W)\hat{\boldsymbol{\epsilon}}_0/(\hat{\boldsymbol{\epsilon}}_0'\hat{\boldsymbol{\epsilon}}_0/NT)\right]^2}{T\mathrm{tr}(W^2 + W'W)} \quad \xrightarrow{\text{d}} \quad \chi^2_1. \tag{30}$$

4.2 Monte-Carlo test procedure

Under H_0 the distributions of the underlying test statistics, basing on the spatially filtered residuals (18), are determined by ρ_0, i.e. (under H_0) a "true" parameter. They are not depending on any nuisance parameters. Hence the test statistics are pivotal and the technique of Monte Carlo (MC) testing can be applied (Dwass, 1957, Barnard, 1963, Dufour and Khalaf, 2001 and Dufour, 2006). This is done, because of two reasons. On the one hand Anselin and Rey (1991) and Florax and Rey (1995) show that the asymptotic distributions of several tests for spatial correlation are sensitive to misspecification of the spatial weights matrices. So, if the weights matrix is misspecified, the GLS estimation (15) and the spatial filtering of $\boldsymbol{X}$ and $\boldsymbol{y}$ can deliver residuals which are not asymptotically distributed as proposed theoretically under (12). On the other hand in small sample sizes the tests' asymptotic distributions are not valid, leading to size distortions, which can be corrected by means of MC-techniques.

The main idea behind the MC method is that critical values and their p-values can be obtained by replacing the "theoretical" null distribution, $F(\mathcal{T}_0)$, through its simulation-based "estimate" (Dufour, 2006), which is given by

$$\widehat{F}_R[\mathcal{T}_0; \mathcal{T}(R)] = \frac{1}{R} \sum_{j=1}^{R} \mathcal{I}_{[0,\infty)}(\mathcal{T}_0 - \mathcal{T}_j), \tag{31}$$

where $\mathcal{T}_0$ is the sample test statistic and $\mathcal{T}(R) = (\mathcal{T}_1, ..., \mathcal{T}_R)'$ are under H_0

simulated test statistics. To simulate these statistics, error terms, $\boldsymbol{u}_j$, are drawn in the j-th replication from the normal distribution with $N(0,1)$. The spatial dependence structure is introduced to the errors for the SAR model by

$$\boldsymbol{e}_j = \left[I_T \otimes (I_N - \theta_0 W)^{-1}\right] \boldsymbol{u}_j \tag{32}$$

and for the SMA model by

$$\boldsymbol{e}_j = \left[I_T \otimes (I_N + \theta_0 W)\right] \boldsymbol{u}_j. \tag{33}$$

The j-th sample is then computed as

$$\boldsymbol{y}_j = \boldsymbol{X}\hat{\boldsymbol{\beta}}_0^{GLS} + \boldsymbol{e}_j, \tag{34}$$

where $\hat{\boldsymbol{\beta}}_0^{GLS}$ is given by (15). The spatially filtered residuals of the j-th simulated sample

$$\hat{\boldsymbol{\epsilon}}_j = \boldsymbol{y}_j^* - \boldsymbol{X}_j^* \hat{\boldsymbol{\beta}}_j^{GLS} \tag{35}$$

enter the test statistic and deliver $\mathcal{T}_j$. The GLS estimator of the regression coefficients of the j-th simulated sample, $\hat{\boldsymbol{\beta}}_j^{GLS}$, is analog to (15). This procedure is done R times, where R is chosen to be sufficiently large. The p-value is obtained by

$$\hat{p}_R(\mathcal{T}_0) = \frac{R\widehat{G}_R(\mathcal{T}_0) + 1}{R+1}, \tag{36}$$

where

$$\widehat{G}_R[\mathcal{T}_0; \boldsymbol{\mathcal{T}}(R)] = \frac{1}{R}\sum_{j=1}^{R} \mathcal{I}_{[0,\infty)}(\mathcal{T}_j - \mathcal{T}_0) \tag{37}$$

is the corresponding sample function of the tail area.

5 Monte Carlo Simulation

5.1 Data Generation

The data generation is closely related to Anselin and Moreno (2003). We consider two different spatial weights matrices, which are created by the rook criterion and a full-distance method. A regular lattice is considered for both weights generations. The rook criterion of contiguity is defined as: $W_{ij} = 1$ for regions that share a common side with the region of interest. The element W_{ij}

of the full-distance weights matrix contains the inverse of the distance between region i and j. The distance is one if they share a common border, two if one have to cross one region to reach from region j to region i, three if there are two regions to cross, etc. However one cannot go diagonally.

We differentiate between two correct and two incorrect spatial weights specifications. When the data is generated by a spatial rook matrix which is also used in the tests or estimation procedure, we call it "correct rook". Accordingly, we call it "correct full" if the full distance weights matrix is used for the data generation and for testing or estimation. The spatial pattern is underspecified when the data is generated by a full distance matrix, whereas we use a rook matrix for estimation and testing. It is over-specified in the opposite case.

The regression part of the model is $\beta_1 + \beta_2 x_{it}$, where β_1 and β_2 are set to 1, the x_{it} are drawn at the beginning from $U[10, 0]$ and are hold fix during the experiment. The errors, u_t, are drawn from $N(0, 1)$. The spatial dependence is introduced for the SAR model by $e_t = (I - \theta W)^{-1} u_t$ and for the SMA by $e_t = (I + \theta W) u_t$. So the dependent variable is computed as

$$y_t = \iota \beta_1 + X_t \beta_2 + e_t,$$

where ι is an $N \times 1$ vector of ones.

We consider three different sample sizes: the small sample case ($T = 5$, $N = 9$), the case with an increased time dimension ($T = 25$, $N = 9$) and the opposite case with an increased cross-sectional dimension ($T = 5$, $N = 25$).

5.2 Small sample properties of correlation tests

The small sample properties are considered for two different null hypotheses. The first one is the usual null when testing for spatial correlation. It tests, whether there is correlation, or not and is denoted as $H_0^1 : \theta = \theta_0 = 0$. The second null tests for remaining spatial correlation in the case if the spatial correlation is filtered out for $\theta_0 = 0.3$ and is denoted as $H_0^2 : \theta = \theta_0 = 0.3$. We run 5,000 replications for each experiment. The number of replications in the MC tests is 999.

5.2.1 Size of the tests

The empirical rejection frequencies under the null hypotheses H_0^1 and H_0^2 are provided by table 1 for the SAR and by table 2 for the SMA model. In the case of testing for $H_0^1 : \theta = \theta_0 = 0$, we only consider the correct weights specifications, because of the absence of spatial correlation in the data generation ($\theta = 0$). This also means that there are no differences between the SAR and SMA model. The results show that the asymptotic versions of almost all tests under-rejects for all sample sizes the null significantly. Except the CD^W and LM^E test for the rook specification. The size distortion is corrected by means of the MC procedure, obtaining rejection frequencies which are lying in the 95% confidence interval [0.0438, 0.0562], constructed as $[\alpha \pm 2\sqrt{\alpha(1-\alpha)/5000}]$, $\alpha = 0.05$.

If we consider the case when testing for remaining spatial correlation, $H_0^2 : \theta = \theta_0 = 0.3$, we discriminate between the cases of correct and miss specification of the spatial weights matrix. If the weights matrix is correct specified, the sizes' pattern over the tests is quite similar to the previous case for the SAR and SMA model. Under a misspecified weights matrix all tests have significant size distortions, except the MC version of the CD and, for the case of overspecification, the CD^W test, which is identical to the CD statistic if a full weights matrix is used. Only in the SMA model with an under-specified weights matrix it under-rejects the null significantly for the small sample size and in the SAR model for large N when W is over-specified. The LM^E statistic has the correct size in both models for the small sample size under an over-specified weights matrix. With increasing T or N it over-rejects the null significantly.

5.2.2 Size Adjusted Power of the tests

To make the power of the tests comparable, we compute the rejection frequencies under the alternative with adjusted nominal levels, in such a way that the empirical level for all tests is the same (Lloyd, 2005). We compute the size adjusted power for two alternatives: $H_1^1 : \theta \neq \theta_0 = 0$, with $\theta = 0.3$ and $H_1^2 : \theta \neq \theta_0 = 0.3$, with $\theta = 0$. The same specifications as before are used. The results are given for the SAR model by table 3 and for the SMA model by table 4. Two results are consistent for both models and all tests: there are no differences between the asymptotic and MC versions of the tests, and all statistics face stronger raises in power due to an increase of T rather of N.

At first, the power of the tests under H_1^1 are considered in detail. All tests face higher power properties in the SAR than in the SMA model, whereas the differences between the tests, the weights specifications and the sample sizes are similar. Under a correct rook specification the power of the CD and BP test is clearly dominated by its transformed versions, the CD^W and BP^W test, respectively. The highest power is accomplished by the LM^E test, followed by the CD^W test. The difference becomes smaller under the full distance weights specification. For all specifications under H_1^1 the CD and CD^W statistic outperforms the BP and BP^W test, respectively. Underspecification of the weights matrix leads to a power loss for the LM^E and CD^W statistic, whereas the CD and BP test are unaffected and the power of the BP^W test even slightly increases. In this case, the CD statistic faces the highest power, irrespectively of the sample size. If the weights matrix is over-specified, the LM^E test is the most powerful test, however, it looses its power compared to the correct rook case.

If we testing against H_1^2 under a correct specification, the LM^E statistic has the highest power for the SAR and SMA model for all sample sizes. Under the correct rook weights matrix, all statistics have higher power than for the case of testing against H_1^1, except the CD test looses its power, even completely for small T. Under the full weights matrix the power of all tests decrease compared to H_1^1. If the spatial weights matrix is under-specified, the LM^E statistic remains the most powerful test. In the SMA model, the adjusted rejection frequencies cannot be computed for the MC version of the CD^W test in the case of $T = 25$ and $N = 9$, and for the LM^E statistic for both large sample sizes, because the size distortions are too large to obtain values different from zero. In the case of overspecification the CD test has the highest power for large T. For large N all tests loose its power completely.

5.3 Estimation properties

The MC simulations are run with 1,000 replications. To obtain the confidence intervals and the point estimators for θ, we apply a grid search from $\theta_0 \in (-0.99,\ 0.99)$ in a step of 0.01. The number of replications for the MC tests is the same as above, i.e. 999. The properties of our test based estimator are compared to the corresponding ML estimator of the SAR (e.g. Anselin, 2008 and Elhorst, 2003) or SMA (e.g. Mur and Angulo, 2007) model, respectively. Its interval estimates are truncated upward to 0.99 and downward to -0.99 if the upper or lower bounds exceed or fall below this values, respectively.

5.3.1 Confidence interval estimation

First, we consider the CI estimation properties under correct specified weights matrices. Table 5 and 6 contain the coverage and in brackets the average length of the CI for θ for the SAR and SMA model, respectively. For both models, the test based CI's, applying the MC procedure, show an exact coverage of the desired level, i.e. 95%, and smaller average lengths than the CI's basing on the asymptotic tests. Only the intervals of the MC versions of the BP and BP^W statistic cover the true parameter too often in the case when T is large. Under the correct rook specification and large N the coverage of the MC version of the BP^W test is about 97% and thus exceeds the 95% coverage as well. The CI's basing on the asymptotic versions of the LM^E and the CD^W test do not yield an exact coverage only under the correct full distance weights matrix for the samples with small T. Similar to the results of the power analysis, under a correct rook specification the transformed statistics, the CD^W and BP^W test, obtain on average smaller CI's than the CD and BP test, respectively. In the SAR model the coverage of the ML estimator is exact 95% for all sample sizes and specifications. The average length of the intervals are the smallest for the small sample case under both correct weights matrices and for large N under the correct full distance matrix. In the other scenarios the CI's basing on the LM^E test are the smallest intervals. In the SMA model specification the ML estimator worsen its performance. Only for large N and $\theta = 0$ under the correct rook specification the coverage is 94.7% and can be regarded as exact. For all other cases the coverage is much lower than 95%. The lowest coverage, 77.4%, is obtained under the full distance weights matrix in the small sample case and $\theta = 0$. Whereas in many cases the exact intervals of the LM^E and CD^W test are on average smaller than the ones of the ML estimator.

In the case when the spatial weights matrix is misspecified we only consider the case of $\theta = 0.5$. Table 7 and 8 show the results for the SAR and the SMA model, respectively. The coverage for almost all CI's is far from exact. Only the CI basing on the MC version of the CD test has a coverage of around 95%. Except for two cases: if the weights matrix is over-specified in the SAR model in the small sample, the coverage is 97.4% and if the weights matrix is under-specified in the SMA model in the large T sample, the coverage is 93.5%. The CI's basing on the asymptotic version of the BP test are around 95% in the SAR model. Only for large T and under-specified weights matrix it is far from the desired level. The coverage of the ML estimator is for both models much lower than 95% in the case of underspecification. The lowest value, 2.3%, is obtained for large T in the SAR model. If the matrix is over-specified, the performance of the ML estimator improves. In the SAR model the coverage is around 98% and exact for large N. It is also exact for the small sample size in the SMA model and 97.6% for large N. For large T the CI covers only in 38.9% the true value of θ. In the cases where the ML estimator obtains useful intervals, i.e. its coverage is around 95%, the average length of the ML estimator's CI is much smaller than the intervals based on the MC version of the CD test. However, these intervals are nearly exact and robust for all specifications.

5.3.2 Point estimation

Table 9 and 10 show the standardized root mean squared error ($RMSE$) and in brackets the bias of the estimated θ under the SAR and SMA specification, respectively. The $RMSE$ is divided though the corresponding $RMSE$ of the ML estimator. Under correct specified weights matrices the estimator basing on the LM^E statistic has the smallest $RMSE$ of all test based estimators. However, in the SAR model they all are outperformed from the ML estimator, whereas the bias of the LM^E test based estimator is quite similar to the one of the ML estimator. In the SMA model, there are several situations, where the estimator basing on the LM^E test obtains the smallest $RMSE$. If the weights matrix is misspecified, in the SAR model the ML estimator remains the best performing estimator. Only in the case of underspecification and for large T, the CD and BP test based estimators obtain smaller $RMSE$'s. In the SMA model, the ML estimator is outperformed for all sample sizes by the LM^E test based estimator, irrespectively if the weights matrix is over- or under-specified. For large T all test based estimators have smaller $RMSE$'s than the ML estimator.

6 Conclusion

In this paper, we apply the method, proposed by Dufour (1990), to construct exact confidence sets for the spatial autocorrelation coefficient in linear panel models with SAR or SMA errors (Cliff and Ord, 1973, 1981, Anselin, 1988 and Anselin et al., 2008). We utilize tests for contemporaneous and spatial correlation among the regression disturbances. Furthermore, we develop upon our procedure a test based estimator for the spatial autocorrelation coefficient.

To indicate the best performing test we run simulation experiments under several spatial specifications. For the SAR as well as for the SMA model, results show that almost all tests have distorted sizes, which can be corrected by applying an MC test procedure, while the size adjusted power remains the same. We indicate the LM^E statistic (Burridge, 1980), extended to the pooled regression model (Anselin et al., 2008), as the most powerful test if the spatial weights matrix is correct specified.

The point and the interval estimator are compared in a Monte Carlo simulation to the ML estimator (e.g. Anselin, 1988, Elhorst, 2003 and Mur and Angulo, 2007). Under correct spatial weights specification the coverage of the CI's basing on the MC versions of almost all considered statistics is exact to the desired level for both models, SAR and SMA. Whereas in the SMA model the ML estimator obtains intervals which are far from the desired level. The CI's basing on the LM^E test are the smallest of the test based intervals. Under misspecification of the spatial weights matrix the MC version of the CD test show the best performance and obtain for almost every scenario exact coverage of the CI's. Thus, the CD test is a robust tool for constructing exact CI's, particularly if the time dimension is large. The test based point estimator basing on the LM^E statistic does nearly perform as good as the ML estimator and even better for some cases with SMA errors, particularly if the weights matrix is misspecified.

Finally, we show an alternative to make exact inference for the spatial autocorrelation coefficient in linear panel models, even if the spatial weights matrix is misspecified. For future research it would be of interest to extend the approach to fixed and random effects specifications, as well as to consider a dynamic panel model and models containing spatially lagged dependent variables. Our procedure can also applied to the cross-sectional case by using appropriate spatial correlation tests.

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A Tables

Table 1: Size of Tests [SAR]

			CD		CD^W		BP		BP^W		LM^E	
θ_0	[test; data]	[N; T]	MC	Asy	MC	Asy	MC	Asy	MC	Asy	MC	Asy
0	[÷; r]	[9; 5]	0.0474	**0.0228**	0.0510	0.0528	0.0470	**0.0272**	0.0488	**0.0158**	0.0492	0.0472
		[9; 25]	0.0448	**0.0386**	0.0472	0.0498	0.0528	0.0458	0.0486	**0.0402**	0.0468	0.0482
		[25; 5]	0.0554	**0.0320**	0.0458	0.0478	0.0520	**0.0292**	0.0462	**0.0146**	0.0472	0.0470
	[÷; f]	[9; 5]	0.0474	**0.0228**	0.0474	**0.0228**	0.0470	**0.0272**	0.0470	**0.0272**	0.0486	**0.0290**
		[9; 25]	0.0448	**0.0386**	0.0448	**0.0386**	0.0528	0.0458	0.0528	0.0458	0.0468	**0.0422**
		[25; 5]	0.0554	**0.0320**	0.0554	**0.0320**	0.0520	**0.0292**	0.0520	**0.0292**	0.0496	**0.0332**
0.3	[r; r]	[9; 5]	0.0508	**0.0252**	0.0460	0.0476	0.0482	**0.0226**	0.0522	**0.0130**	0.0500	0.0470
	"correct	[9; 25]	0.0508	0.0442	0.0498	0.0516	0.0468	**0.0412**	0.0502	**0.0402**	0.0516	0.0512
	rook"	[25; 5]	0.0470	**0.0244**	0.0514	0.0516	0.0494	**0.0248**	0.0504	**0.0150**	0.0506	0.0506
	[f; f]	[9; 5]	0.0514	**0.0252**	0.0514	**0.0252**	0.0484	**0.0228**	0.0484	**0.0228**	0.0492	**0.0308**
	"correct	[9; 25]	0.0500	0.0440	0.0500	0.0440	0.0468	**0.0406**	0.0468	**0.0406**	0.0472	0.0440
	full"	[25; 5]	0.0476	**0.0242**	0.0476	**0.0242**	0.0484	**0.0252**	0.0484	**0.0252**	0.0494	**0.0328**
	[r; f]	[9; 5]	0.0504	**0.0256**	**0.1508**	**0.1546**	**0.0614**	**0.0302**	**0.0732**	**0.0194**	**0.1674**	**0.1616**
	"under-	[9; 25]	0.0504	**0.0432**	**0.4280**	**0.4330**	**0.1378**	**0.1214**	**0.1714**	**0.1514**	**0.4558**	**0.4594**
	spec"	[25; 5]	0.0466	**0.0240**	**0.3888**	**0.3912**	**0.0600**	**0.0362**	**0.0948**	**0.0324**	**0.4616**	**0.4618**
	[f; r]	[9; 5]	0.0452	**0.0216**	0.0452	**0.0216**	**0.0676**	**0.0306**	**0.0676**	**0.0306**	0.0452	**0.0320**
	"over-	[9; 25]	0.0472	**0.0422**	0.0472	**0.0422**	**0.1352**	**0.1214**	**0.1352**	**0.1214**	**0.0806**	**0.0768**
	spec"	[25; 5]	**0.0404**	**0.0186**	**0.0404**	**0.0186**	**0.0642**	**0.0382**	**0.0642**	**0.0382**	**0.1018**	**0.0768**

Notes: 5,000 replications; 999 MC Replications; size of bold numbers lie beyond the interval $[\alpha \pm 2\sqrt{\alpha(1-\alpha)/5000}]$; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix and ÷ that the use of a weight matrix is irrelevant

Table 2: Size of Tests [SMA]

			CD		CD^W		BP		BP^W		LM^E	
θ_0	[test; data]	[N; T]	MC	Asy	MC	Asy	MC	Asy	MC	Asy	MC	Asy
0	[÷; r]	[9; 5]	0.0474	**0.0228**	0.0522	0.0528	0.0478	**0.0272**	0.0466	**0.0158**	0.0498	0.0472
		[9; 25]	0.0444	**0.0386**	0.0494	0.0498	0.0518	0.0458	0.0488	**0.0402**	0.0478	0.0482
		[25; 5]	0.0568	**0.0320**	0.0476	0.0478	0.0514	**0.0292**	0.0464	**0.0146**	0.0486	0.0470
	[÷; f]	[9; 5]	0.0474	**0.0228**	0.0474	**0.0228**	0.0478	**0.0272**	0.0478	**0.0272**	0.0472	**0.0290**
		[9; 25]	0.0444	**0.0386**	0.0444	**0.0386**	0.0518	0.0458	0.0518	0.0458	0.0450	**0.0422**
		[25; 5]	0.0568	**0.0320**	0.0568	**0.0320**	0.0514	**0.0292**	0.0514	**0.0292**	0.0514	**0.0332**
0.3	[r; r]	[9; 5]	0.0460	**0.0242**	0.0504	0.0514	0.0516	**0.0282**	0.0552	**0.0176**	0.0480	**0.0436**
	"correct	[9; 25]	0.0532	0.0456	0.0514	0.0506	0.0494	**0.0430**	0.0476	**0.0394**	0.0542	0.0550
	rook"	[25; 5]	0.0492	**0.0264**	0.0512	0.0500	0.0472	**0.0248**	0.0444	**0.0146**	0.0478	0.0468
	[f; f]	[9; 5]	0.0464	**0.0248**	0.0464	**0.0248**	0.0508	**0.0276**	0.0508	**0.0276**	0.0452	**0.0288**
	"correct	[9; 25]	0.0530	0.0460	0.0530	0.0460	0.0500	**0.0430**	0.0500	**0.0430**	0.0526	0.0492
	full"	[25; 5]	0.0486	**0.0258**	0.0486	**0.0258**	0.0476	**0.0266**	0.0476	**0.0266**	0.0446	**0.0320**
	[r; f]	[9; 5]	**0.0376**	**0.0194**	**0.1954**	**0.2008**	**0.0734**	**0.0408**	**0.0826**	**0.0302**	**0.2178**	**0.2108**
	"under-	[9; 25]	0.0480	**0.0396**	**0.5886**	**0.5918**	**0.1954**	**0.1824**	**0.2514**	**0.2264**	**0.6070**	**0.6078**
	spec"	[25; 5]	0.0438	**0.0214**	**0.4700**	**0.4724**	**0.0684**	**0.0384**	**0.0990**	**0.0368**	**0.5498**	**0.5478**
	[f; r]	[9; 5]	0.0484	**0.0246**	0.0484	**0.0246**	**0.0646**	**0.0326**	**0.0646**	**0.0326**	0.0452	**0.0330**
	"over-	[9; 25]	0.0542	0.0468	0.0542	0.0468	**0.1430**	**0.1284**	**0.1430**	**0.1284**	**0.1124**	**0.1070**
	spec"	[25; 5]	0.0456	**0.0252**	0.0456	**0.0252**	**0.0606**	**0.0334**	**0.0606**	**0.0334**	**0.1048**	**0.0808**

Notes: 5,000 replications; 999 MC Replications; size of bold numbers lie beyond the interval $[\alpha \pm 2\sqrt{\alpha(1-\alpha)/5000}]$; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix and ÷ that the use of a weight matrix is irrelevant

Table 3: Size Adjusted Power of Tests [SAR]

			CD		CD^W		BP		BP^W		LM^E	
$[\theta_0; \theta]$	[test; data]	[N; T]	MC	Asy	MC	Asy	MC	Asy	MC	Asy	MC	Asy
[0; 0.3]	[r; r]	[9; 5]	0.2498	0.2532	0.2598	0.2566	0.0886	0.0900	0.1094	0.1122	0.3174	0.3170
	"correct	[9; 25]	0.7118	0.7154	0.9662	0.9654	0.4584	0.4680	0.7308	0.7306	0.9744	0.9748
	rook"	[25; 5]	0.2314	0.2348	0.6416	0.6420	0.0740	0.0770	0.1502	0.1508	0.7382	0.7356
	[f; f]	[9; 5]	0.2752	0.2772	0.2752	0.2772	0.0716	0.0744	0.0716	0.0744	0.2562	0.2602
	"correct	[9; 25]	0.7694	0.7724	0.7694	0.7724	0.3026	0.3116	0.3026	0.3116	0.8062	0.8102
	full"	[25; 5]	0.2650	0.2676	0.2650	0.2676	0.0484	0.0512	0.0484	0.0512	0.2792	0.2830
	[r; f]	[9; 5]	0.2752	0.2772	0.1474	0.1450	0.0716	0.0744	0.0752	0.0754	0.1550	0.1556
	"under-	[9; 25]	0.7694	0.7724	0.6750	0.6742	0.3026	0.3116	0.3156	0.3182	0.6882	0.6894
	spec"	[25; 5]	0.2650	0.2676	0.1472	0.1490	0.0484	0.0512	0.0654	0.0642	0.1608	0.1610
	[f; r]	[9; 5]	0.2498	0.2532	0.2498	0.2532	0.0886	0.0900	0.0886	0.0900	0.2998	0.3044
	"over-	[9; 25]	0.7118	0.7154	0.7118	0.7154	0.4584	0.4680	0.4584	0.4680	0.9114	0.9138
	spec"	[25; 5]	0.2314	0.2348	0.2314	0.2348	0.0740	0.0770	0.0740	0.0770	0.4850	0.4934
[0.3; 0]	[r; r]	[9; 5]	0.0172	0.0214	0.3556	0.3576	0.0896	0.0880	0.1094	0.1126	0.4010	0.4028
	"correct	[9; 25]	0.5886	0.5876	0.9614	0.9630	0.4380	0.4316	0.7016	0.7046	0.9716	0.9706
	rook"	[25; 5]	0.0244	0.0208	0.6534	0.6510	0.0830	0.0834	0.1416	0.1370	0.7422	0.7408
	[f; f]	[9; 5]	0.0184	0.0182	0.0184	0.0182	0.0594	0.0624	0.0594	0.0624	0.1128	0.1058
	"correct	[9; 25]	0.5724	0.5658	0.5724	0.5658	0.1406	0.1388	0.1406	0.1388	0.7032	0.6978
	full"	[25; 5]	0.0220	0.0204	0.0220	0.0204	0.0590	0.0596	0.0590	0.0596	0.1390	0.1358
	[r; f]	[9; 5]	0.0172	0.0246	0.1454	0.1500	0.0732	0.0740	0.0822	0.0862	0.1608	0.1578
	"under-	[9; 25]	0.5840	0.5900	0.6054	0.5426	0.2358	0.2358	0.4162	0.4314	0.6476	0.5746
	spec"	[25; 5]	0.0244	0.0184	0.1402	0.1288	0.0676	0.0700	0.0824	0.0846	0.2138	0.1618
	[f; r]	[9; 5]	0.0270	0.0312	0.0270	0.0312	0.0462	0.0470	0.0462	0.0470	0.1244	0.1222
	"over-	[9; 25]	0.5902	0.5866	0.5902	0.5866	0.0424	0.0436	0.0424	0.0436	0.5446	0.5398
	spec"	[25; 5]	0.0386	0.0376	0.0386	0.0376	0.0428	0.0412	0.0428	0.0412	0.0236	0.0216

Notes: 5,000 replications; 999 MC Replications; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix and ÷ that the use of a weight matrix is irrelevant

Table 4: Size Adjusted Power of Tests [SMA]

			CD		CD^W		BP		BP^W		LM^E	
$[\theta_0; \theta]$	[test; data]	[N; T]	MC	Asy	MC	Asy	MC	Asy	MC	Asy	MC	Asy
[0; 0.3]	[r; r]	[9; 5]	0.1818	0.1826	0.2108	0.2084	0.0858	0.0900	0.0952	0.0956	0.2618	0.2594
	"correct	[9; 25]	0.5384	0.5412	0.9528	0.9526	0.3780	0.3914	0.6670	0.6696	0.9634	0.9640
	rook"	[25; 5]	0.1808	0.1834	0.5884	0.5886	0.0642	0.0650	0.1306	0.1304	0.6804	0.6814
	[f; f]	[9; 5]	0.1874	0.1890	0.1874	0.1890	0.0634	0.0642	0.0634	0.0642	0.1788	0.1826
	"correct	[9; 25]	0.5654	0.5682	0.5654	0.5682	0.1670	0.1762	0.1670	0.1762	0.6316	0.6386
	full"	[25; 5]	0.1920	0.1934	0.1920	0.1934	0.0464	0.0444	0.0464	0.0444	0.2040	0.2064
	[r; f]	[9; 5]	0.1874	0.1890	0.1048	0.1042	0.0634	0.0642	0.0650	0.0658	0.1134	0.1160
	"under-	[9; 25]	0.5654	0.5682	0.5206	0.5178	0.1670	0.1762	0.2020	0.2030	0.5326	0.5352
	spec"	[25; 5]	0.1920	0.1934	0.1188	0.1170	0.0464	0.0444	0.0530	0.0544	0.1288	0.1268
	[f; r]	[9; 5]	0.1818	0.1826	0.1818	0.1826	0.0858	0.0900	0.0858	0.0900	0.2242	0.2270
	"over-	[9; 25]	0.5384	0.5412	0.5384	0.5412	0.3780	0.3914	0.3780	0.3914	0.8296	0.8338
	spec"	[25; 5]	0.1808	0.1834	0.1808	0.1834	0.0642	0.0650	0.0642	0.0650	0.3814	0.3840
[0.3; 0]	[r; r]	[9; 5]	0.0300	0.0298	0.3982	0.3968	0.1010	0.0976	0.1292	0.1244	0.4658	0.4698
	"correct	[9; 25]	0.4132	0.4292	0.9730	0.9728	0.5130	0.5150	0.7668	0.7680	0.9792	0.9798
	rook"	[25; 5]	0.0152	0.0136	0.6942	0.7020	0.0914	0.0940	0.1698	0.1684	0.7864	0.7860
	[f; f]	[9; 5]	0.0254	0.0244	0.0254	0.0244	0.0564	0.0548	0.0564	0.0548	0.1056	0.0982
	"correct	[9; 25]	0.3326	0.3430	0.3326	0.3430	0.1148	0.1166	0.1148	0.1166	0.5184	0.5174
	full"	[25; 5]	0.0164	0.0154	0.0164	0.0154	0.0558	0.0582	0.0558	0.0582	0.1250	0.1230
	[r; f]	[9; 5]	0.0450	0.0426	0.1322	0.1410	0.0690	0.0688	0.0880	0.0838	0.1474	0.1524
	"under-	[9; 25]	0.4484	0.4522	-	0.5362	0.2302	0.2198	0.3936	0.4110	-	0.5646
	spec"	[25; 5]	0.0250	0.0212	0.1906	0.1348	0.0642	0.0688	0.0874	0.0870	-	0.1696
	[f; r]	[9; 5]	0.0210	0.0218	0.0210	0.0218	0.0452	0.0420	0.0452	0.0420	0.1092	0.1046
	"over-	[9; 25]	0.3272	0.3424	0.3272	0.3424	0.0332	0.0312	0.0332	0.0312	0.2482	0.2532
	spec"	[25; 5]	0.0186	0.0194	0.0186	0.0194	0.0424	0.0434	0.0424	0.0434	0.0174	0.0184

Notes: 5,000 replications; 999 MC Replications; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix and ÷ that the use of a weight matrix is irrelevant

Table 5: Coverage and average length of 95% Confidence Intervals of θ (Correct Specified) [SAR]

			CD		CD^W		BP		BP^W		LM^E		ML
[estim; data]	[N; T]	θ	MC	Asy	MC	Asy	MC	Asy	MC	Asy	MC	Asy	
[r; r]	[9; 5]	0	0.9490	0.9770	0.9460	0.9470	0.9468	0.9789	0.9515	0.9900	0.9540	0.9580	0.9420
“correct			[1.5281]	[1.7503]	[0.8966]	[0.8980]	[1.6155]	[1.7621]	[1.3830]	[1.6439]	[0.7997]	[0.8146]	[0.7323]
rook”		0.5	0.9490	0.9760	0.9480	0.9450	0.9496	0.9779	0.9545	0.9900	0.9500	0.9580	0.9530
			[1.0536]	[1.1967]	[0.7222]	[0.7244]	[1.3725]	[1.5219]	[1.0991]	[1.3225]	[0.6576]	[0.6711]	[0.5986]
	[9;25]	0	0.9530	0.9580	0.9470	0.9450	0.9743	0.9917	0.9751	0.9886	0.9440	0.9500	0.9500
			[0.5717]	[0.5938]	[0.3092]	[0.3076]	[0.6527]	[0.6774]	[0.4932]	[0.5120]	[0.2988]	[0.2993]	[0.3230]
		0.5	0.9520	0.9580	0.9460	0.9470	0.9814	0.9938	0.9772	0.9875	0.9460	0.9500	0.9570
			[0.3133]	[0.3228]	[0.2495]	[0.2480]	[0.5204]	[0.5382]	[0.4060]	[0.4204]	[0.2430]	[0.2428]	[0.2499]
	[25; 5]	0	0.9480	0.9720	0.9420	0.9430	0.9575	0.9839	0.9702	0.9889	0.9500	0.9500	0.9480
			[1.4957]	[1.7372]	[0.5150]	[0.5149]	[1.6492]	[1.7658]	[1.1337]	[1.3622]	[0.4567]	[0.4591]	[0.4735]
		0.5	0.9510	0.9710	0.9420	0.9400	0.9604	0.9848	0.9763	0.9899	0.9470	0.9500	0.9590
			[1.0223]	[1.1784]	[0.4319]	[0.4331]	[1.3434]	[1.4715]	[0.8853]	[1.0260]	[0.3831]	[0.3856]	[0.3835]
[f; f]	[9; 5]	0	0.9480	0.9770	0.9480	0.9770	0.9447	0.9779	0.9447	0.9779	0.9460	0.9810	0.9490
“correct			[1.5042]	[1.7237]	[1.5042]	[1.7237]	[1.7975]	[1.8936]	[1.7975]	[1.8936]	[1.2988]	[1.4659]	[1.1945]
full”		0.5	0.9460	0.9750	0.9460	0.9750	0.9467	0.9760	0.9467	0.9760	0.9620	0.9780	0.9400
			[0.9246]	[1.0579]	[0.9246]	[1.0579]	[1.3854]	[1.4935]	[1.3854]	[1.4935]	[0.8620]	[0.9506]	[0.8247]
	[9;25]	0	0.9530	0.9580	0.9530	0.9580	0.9752	0.9917	0.9752	0.9917	0.9450	0.9560	0.9480
			[0.5521]	[0.5723]	[0.5521]	[0.5723]	[1.1738]	[1.2247]	[1.1738]	[1.2247]	[0.4956]	[0.5061]	[0.5068]
		0.5	0.9530	0.9580	0.9530	0.9580	0.9784	0.9886	0.9784	0.9886	0.9510	0.9540	0.9560
			[0.2899]	[0.2989]	[0.2899]	[0.2989]	[0.6704]	[0.6940]	[0.6704]	[0.6940]	[0.2872]	[0.2925]	[0.3048]
	[25; 5]	0	0.9480	0.9720	0.9480	0.9720	0.9584	0.9889	0.9584	0.9889	0.9440	0.9640	0.9490
			[1.4819]	[1.7013]	[1.4819]	[1.7013]	[1.8528]	[1.9216]	[1.8528]	[1.9216]	[1.1714]	[1.3019]	[1.1197]
		0.5	0.9490	0.9710	0.9490	0.9710	0.9623	0.9879	0.9623	0.9879	0.9480	0.9660	0.9440
			[0.8741]	[0.9997]	[0.8741]	[0.9997]	[1.5123]	[1.5947]	[1.5123]	[1.5947]	[0.7662]	[0.8348]	[0.7407]

Notes: 1,000 replications; estimated with grid of 0.01 and $R_{MC} = 999$; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix; numbers in brackets denotes the average length of the confidence intervals

Table 6: Coverage and average length of 95% Confidence Intervals of θ (Correct Specified) [SMA]

			CD		CD^W		BP		BP^W		LM^E		ML
[estim; data]	[N; T]	θ	MC	Asy	MC	Asy	MC	Asy	MC	Asy	MC	Asy	
[r; r]	[9; 5]	0	0.9490	0.9770	0.9460	0.9470	0.9437	0.9760	0.9505	0.9900	0.9540	0.9580	0.9030
"correct			[1.3548]	[1.5196]	[0.7637]	[0.7628]	[1.2035]	[1.3058]	[1.0710]	[1.2313]	[0.7077]	[0.7171	[0.7675]
rook"		0.5	0.9420	0.9760	0.9500	0.9450	0.9496	0.9749	0.9516	0.9910	0.9510	0.9580	0.8890
			[1.1582]	[1.2559]	[0.6781]	[0.6808]	[1.1962]	[1.3090]	[1.0095]	[1.1992]	[0.6195]	[0.6322	[0.5558]
	[9;25]	0	0.9530	0.9580	0.9470	0.9450	0.9753	0.9907	0.9751	0.9886	0.9440	0.9500	0.9470
			[0.5305]	[0.5517]	[0.3018]	[0.3002]	[0.5850]	[0.6035]	[0.4688]	[0.4850]	[0.2923]	[0.2928	[0.3226]
		0.5	0.9520	0.9580	0.9500	0.9480	0.9793	0.9897	0.9730	0.9855	0.9490	0.9500	0.9320
			[0.5549]	[0.5709]	[0.2473]	[0.2465]	[0.4975]	[0.5128]	[0.3940]	[0.4073]	[0.2409]	[0.2411	[0.2374]
	[25; 5]	0	0.9470	0.9720	0.9420	0.9430	0.9595	0.9849	0.9702	0.9889	0.9500	0.9500	0.9320
			[1.2826]	[1.4912]	[0.4854]	[0.4856]	[1.2289]	[1.3175]	[0.9130]	[1.0466]	[0.4367]	[0.4388	[0.4848]
		0.5	0.9470	0.9700	0.9400	0.9410	0.9554	0.9828	0.9733	0.9870	0.9480	0.9510	0.9140
			[1.1279]	[1.2415]	[0.4179]	[0.4197]	[1.1831]	[1.2822]	[0.8313]	[0.9643]	[0.3744]	[0.3774	[0.3623]
[f; f]	[9; 5]	0	0.9490	0.9770	0.9490	0.9770	0.9448	0.9760	0.9448	0.9760	0.9460	0.9810	0.7740
"correct			[1.3791]	[1.4831]	[1.3791]	[1.4831]	[1.5521]	[1.6356]	[1.5521]	[1.6356]	[1.1767]	[1.3122	[1.0413]
full"		0.5	0.9419	0.9750	0.9419	0.9750	0.9475	0.9769	0.9475	0.9769	0.9540	0.9820	0.8400
			[1.1074]	[1.1820]	[1.1074]	[1.1820]	[1.3998]	[1.4732]	[1.3998]	[1.4732]	[1.0616]	[1.1270	[1.1744]
	[9;25]	0	0.9530	0.9580	0.9530	0.9580	0.9732	0.9907	0.9732	0.9907	0.9450	0.9560	0.9180
			[0.5687]	[0.5967]	[0.5687]	[0.5967]	[1.0673]	[1.1087]	[1.0673]	[1.1087]	[0.4879]	[0.5001	[0.4887]
		0.5	0.9530	0.9580	0.9530	0.9580	0.9783	0.9917	0.9783	0.9917	0.9500	0.9540	0.9210
			[0.6898]	[0.7125]	[0.6898]	[0.7125]	[0.9355]	[0.9584]	[0.9355]	[0.9584]	[0.5907]	[0.6031	[0.5804]
	[25; 5]	0	0.9480	0.9720	0.9480	0.9720	0.9574	0.9879	0.9574	0.9879	0.9440	0.9640	0.8300
			[1.3416]	[1.4396]	[1.3416]	[1.4396]	[1.6115]	[1.6772]	[1.6115]	[1.6772]	[1.0917]	[1.2048	[1.0203]
		0.5	0.9518	0.9730	0.9518	0.9730	0.9612	0.9898	0.9612	0.9898	0.9479	0.9660	0.8700
			[1.0761]	[1.1479]	[1.0761]	[1.1479]	[1.4602]	[1.5215]	[1.4602]	[1.5215]	[0.9957]	[1.0469	[1.1124]

Notes: 1,000 replications; estimated with grid of 0.01 and $R_{MC} = 999$; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix; numbers in brackets denotes the average length of the confidence intervals

Table 7: Coverage and average length of 95% Confidence Intervals of θ (Miss-Specified) [SAR]

			CD		CD^W		BP		BP^W		LM^E		ML
[estim; data]	[N; T]	θ	MC	Asy	MC	Asy	MC	Asy	MC	Asy	MC	Asy	
[r; f]	[9; 5]	0.5	0.9390	0.9750	0.6980	0.7010	0.9163	0.9508	0.8924	0.9649	0.6800	0.6910	0.7520
“under-			[0.9485]	[1.0846]	[0.6613]	[0.6619]	[1.3052]	[1.4454]	[1.0892]	[1.3211]	[0.6053]	[0.6166]	[0.6345]
spec”	[9;25]	0.5	0.9520	0.9580	0.2080	0.2080	0.7417	0.7764	0.5797	0.6050	0.1690	0.1660	0.4000
			[0.2883]	[0.2963]	[0.2206]	[0.2194]	[0.3641]	[0.3810]	[0.3655]	[0.3772]	[0.2167]	[0.2165]	[0.2595]
	[25; 5]	0.5	0.9530	0.9760	0.2340	0.2380	0.9138	0.9585	0.8060	0.9215	0.1490	0.1500	0.2430
			[0.8765]	[1.0191]	[0.4235]	[0.4243]	[1.3909]	[1.5124]	[0.9499]	[1.1329]	[0.3808]	[0.3826]	[0.4348]
[f; r]	[9; 5]	0.5	0.9740	0.9840	0.9740	0.9840	0.8956	0.9577	0.8956	0.9577	0.9590	0.9720	0.9820
“over-			[1.0036]	[1.1457]	[1.0036]	[1.1457]	[1.3779]	[1.5204]	[1.3779]	[1.5204]	[0.9217]	[0.9781]	[0.8127]
spec”	[9;25]	0.5	0.9490	0.9580	0.9490	0.9580	0.8654	0.9427	0.8654	0.9427	0.8600	0.8610	0.9810
			[0.3134]	[0.3231]	[0.3134]	[0.3231]	[0.5440]	[0.6066]	[0.5440]	[0.6066]	[0.4095]	[0.4193]	[0.3077]
	[25; 5]	0.5	0.9600	0.9840	0.9600	0.9840	0.9203	0.9609	0.9203	0.9609	0.6980	0.7380	0.9500
			[0.9722]	[1.1145]	[0.9722]	[1.1145]	[1.4329]	[1.5548]	[1.4329]	[1.5548]	[0.6242]	[0.6609]	[0.6645]

Notes: 1,000 replications; estimated with grid of 0.01 and $R_{MC} = 999$; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix; numbers in brackets denotes the average length of the confidence intervals

Table 8: Coverage and average length of 95% Confidence Intervals of θ (Miss-Specified) [SMA]

			CD		CD^W		BP		BP^W		LM^E		ML
[estim; data]	[N; T]	θ	MC	Asy	MC	Asy	MC	Asy	MC	Asy	MC	Asy	
[r; f]	[9; 5]	0.5	0.9650	0.9860	0.5200	0.5250	0.8434	0.9044	0.8115	0.9089	0.4640	0.4790	0.5840
"under-			[1.1135]	[1.2176]	[0.6598]	[0.6599]	[1.1092]	[1.2056]	[0.9804]	[1.1388]	[0.6170]	[0.6262]	[0.6757]
spec"	[9;25]	0.5	0.9350	0.9450	0.0260	0.0270	0.2487	0.2647	0.1767	0.1944	0.0200	0.0190	0.0230
			[0.4677]	[0.4824]	[0.2488]	[0.2475]	[0.4341]	[0.4495]	[0.4000]	[0.4131]	[0.2439]	[0.2438]	[0.2692]
	[25; 5]	0.5	0.9630	0.9850	0.1140	0.1140	0.8545	0.9159	0.6643	0.8147	0.0650	0.0650	0.1010
			[1.0518]	[1.1894]	[0.4383]	[0.4386]	[1.1333]	[1.2180]	[0.8482]	[0.9750]	[0.3959]	[0.3979]	[0.4350]
[f; r]	[9; 5]	0.5	0.9540	0.9780	0.9540	0.9780	0.9062	0.9605	0.9062	0.9605	0.9299	0.9549	0.9520
"over-			[1.1357]	[1.2109]	[1.1357]	[1.2109]	[1.3452]	[1.4546]	[1.3452]	[1.4546]	[1.0303]	[1.0782]	[1.3336]
spec"	[9;25]	0.5	0.9510	0.9600	0.9510	0.9600	0.7290	0.8058	0.7290	0.8058	0.7680	0.7810	0.3890
			[0.7252]	[0.7479]	[0.7252]	[0.7479]	[0.6546]	[0.7192]	[0.6546]	[0.7192]	[0.6009]	[0.6081]	[0.5066]
	[25; 5]	0.5	0.9458	0.9730	0.9458	0.9730	0.9226	0.9762	0.9226	0.9762	0.7862	0.8278	0.9760
			[1.1023]	[1.1788]	[1.1023]	[1.1788]	[1.3881]	[1.4849]	[1.3881]	[1.4849]	[0.7605]	[0.8082]	[1.0392]

Notes: 1,000 replications; estimated with grid of 0.01 and $R_{MC} = 999$; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix; numbers in brackets denotes the average length of the confidence intervals

Table 9: RMSE(θ) and BIAS(θ) [SAR]

[estim; data]	[N; T]	θ	CD	CD^W	BP	BP^W	LM^E	ML
[r; r]	[9; 5]	0	3.0814	1.1717	2.3965	1.6877	1.0866	1.0000
“correct			[-0.3146]	[-0.0358]	[-0.0191]	[-0.0517]	[-0.0382]	[-0.0354]
rook”		0.5	2.8979	1.1290	2.2251	1.6133	1.0482	1.0000
			[-0.2294]	[-0.0330]	[-0.0416]	[-0.0319]	[-0.0332]	[-0.0581]
	[9;25]	0	2.1719	1.0599	1.1890	1.0615	1.0160	1.0000
			[-0.0547]	[-0.0093]	[-0.0086]	[-0.0092]	[-0.0092]	[-0.0090]
		0.5	1.5038	1.0878	1.2091	1.1070	1.0504	1.0000
			[-0.0299]	[-0.0081]	[-0.0157]	[-0.0093]	[-0.0077]	[-0.0123]
	[25; 5]	0	4.4785	1.1406	3.6774	1.5844	1.0302	1.0000
			[-0.2807]	[-0.0115]	[0.0004]	[-0.0084]	[-0.0120]	[-0.0117]
		0.5	4.6783	1.1639	3.6157	1.7303	1.0541	1.0000
			[-0.2175]	[-0.0102]	[-0.0173]	[0.0000]	[-0.0099]	[-0.0201]
[f; f]	[9; 5]	0	1.3482	1.3482	1.6633	1.6633	1.0973	1.0000
“correct			[-0.3071]	[-0.3071]	[-0.0490]	[-0.0490]	[-0.2246]	[-0.2021]
full”		0.5	1.3153	1.3153	1.7114	1.7114	1.0247	1.0000
			[-0.1899]	[-0.1899]	[-0.1364]	[-0.1364]	[-0.1407]	[-0.1483]
	[9;25]	0	1.1886	1.1886	1.2554	1.2554	1.0132	1.0000
			[-0.0514]	[-0.0514]	[-0.0493]	[-0.0493]	[-0.0373]	[-0.0369]
		0.5	1.0654	1.0654	1.2619	1.2619	1.0067	1.0000
			[-0.0270]	[-0.0270]	[-0.0357]	[-0.0357]	[-0.0221]	[-0.0244]
	[25; 5]	0	1.4654	1.4654	2.1805	2.1805	1.0657	1.0000
			[-0.2614]	[-0.2614]	[0.0186]	[0.0186]	[-0.1549]	[-0.1462]
		0.5	1.4445	1.4445	2.1278	2.1278	1.0354	1.0000
			[-0.1705]	[-0.1705]	[-0.0760]	[-0.0760]	[-0.1037]	[-0.1157]
[r; f]	[9; 5]	0.5	1.3504	1.0363	1.4149	1.2070	1.0212	1.0000
“under-			[-0.1917]	[-0.2226]	[-0.2036]	[-0.2358]	[-0.2279]	[-0.2144]
spec”	[9;25]	0.5	0.4970	1.1260	0.9465	1.1548	1.1413	1.0000
			[-0.0266]	[-0.1745]	[-0.1346]	[-0.1800]	[-0.1785]	[-0.1492]
	[25; 5]	0.5	1.1030	1.0255	1.3502	1.0802	1.0231	1.0000
			[-0.1760]	[-0.3081]	[-0.2559]	[-0.3108]	[-0.3118]	[-0.3007]
[f; r]	[9; 5]	0.5	2.0886	2.0886	2.4503	2.4503	1.0769	1.0000
“over-			[-0.2374]	[-0.2374]	[-0.0464]	[-0.0464]	[-0.0018]	[-0.0821]
spec”	[9;25]	0.5	1.6570	1.6570	3.1389	3.1389	1.6559	1.0000
			[-0.0583]	[-0.0583]	[0.0449]	[0.0449]	[0.0806]	[0.0103]
	[25; 5]	0.5	3.2051	3.2051	3.9188	3.9188	2.7580	1.0000
			[-0.2359]	[-0.2359]	[0.1014]	[0.1014]	[0.3112]	[0.0638]

Notes: 1,000 replications; estimated with grid of 0.01 and $R_{MC} = 999$; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix; numbers in brackets denotes the average length of the confidence intervals

Table 10: RMSE(θ) and BIAS(θ) [SMA]

[estim; data]	[N; T]	θ	CD	CD^W	BP	BP^W	LM^E	ML
[r; r]	[9; 5]	0	1.8922	0.9888	1.7394	1.2722	0.9160	1.0000
“correct			[-0.1966]	[-0.0374]	[-0.0393]	[-0.0502]	[-0.0393]	[-0.0423]
rook”		0.5	3.2956	1.0761	2.1879	1.5065	1.0094	1.0000
			[-0.2984]	[-0.0358]	[-0.0830]	[-0.0551]	[-0.0350]	[-0.0132]
	[9;25]	0	1.8174	1.0303	1.2057	1.0372	0.9877	1.0000
			[-0.0381]	[-0.0094]	[-0.0091]	[-0.0094]	[-0.0091]	[-0.0093]
		0.5	2.9018	1.1001	1.2298	1.1184	1.0587	1.0000
			[-0.0593]	[-0.0080]	[-0.0017]	[-0.0062]	[-0.0076]	[-0.0031]
	[25; 5]	0	3.0096	1.0687	2.6817	1.4233	0.9680	1.0000
			[-0.1715]	[-0.0118]	[0.0041]	[-0.0097]	[-0.0121]	[-0.0125]
		0.5	5.3309	1.1410	3.3758	1.5718	1.0344	1.0000
			[-0.2835]	[-0.0108]	[-0.0755]	[-0.0153]	[-0.0099]	[-0.0004]
[f; f]	[9; 5]	0	1.0795	1.0795	1.5468	1.5468	0.9369	1.0000
“correct			[-0.1680]	[-0.1680]	[-0.0318]	[-0.0318]	[-0.1401]	[-0.1431]
full”		0.5	1.1523	1.1523	1.3094	1.3094	0.9519	1.0000
			[-0.2399]	[-0.2399]	[-0.2086]	[-0.2086]	[-0.1771]	[-0.1594]
	[9;25]	0	1.0970	1.0970	1.2799	1.2799	0.9839	1.0000
			[-0.0325]	[-0.0325]	[-0.0325]	[-0.0325]	[-0.0258]	[-0.0259]
		0.5	1.2097	1.2097	1.2082	1.2082	1.0018	1.0000
			[-0.0464]	[-0.0464]	[-0.0355]	[-0.0355]	[-0.0335]	[-0.0273]
	[25; 5]	0	1.1615	1.1615	2.0496	2.0496	0.9405	1.0000
			[-0.1348]	[-0.1348]	[0.0869]	[0.0869]	[-0.0915]	[-0.0921]
		0.5	1.2587	1.2587	1.5232	1.5232	0.9592	1.0000
			[-0.2214]	[-0.2214]	[-0.1355]	[-0.1355]	[-0.1289]	[-0.1133]
[r; f]	[9; 5]	0.5	1.3831	1.0050	1.2674	1.1096	0.9902	1.0000
“under-			[-0.3164]	[-0.2991]	[-0.3210]	[-0.3127]	[-0.3020]	[-0.3047]
spec”	[9;25]	0.5	0.6215	0.9570	0.9781	0.9624	0.9615	1.0000
			[-0.1157]	[-0.2597]	[-0.2642]	[-0.2611]	[-0.2619]	[-0.2746]
	[25; 5]	0.5	1.2303	0.9980	1.2054	1.0368	0.9925	1.0000
			[-0.3051]	[-0.3599]	[-0.3577]	[-0.3615]	[-0.3617]	[-0.3655]
[f; r]	[9; 5]	0.5	1.1724	1.1724	1.3110	1.3110	0.8142	1.0000
“over-			[-0.2530]	[-0.2530]	[-0.0868]	[-0.0868]	[0.0789]	[0.2116]
spec”	[9;25]	0.5	0.4856	0.4856	0.9838	0.9838	0.6834	1.0000
			[-0.0521]	[-0.0521]	[0.3734]	[0.3734]	[0.2480]	[0.4141]
	[25; 5]	0.5	1.0791	1.0791	1.2350	1.2350	0.8893	1.0000
			[-0.2430]	[-0.2430]	[0.0349]	[0.0349]	[0.3524]	[0.4437]

Notes: 1,000 replications; estimated with grid of 0.01 and $R_{MC} = 999$; f denotes the use of a full distance weight matrix, r the use of a rook weight matrix; numbers in brackets denotes the average length of the confidence intervals