A spatially augmented Mankiw-Romer-Weil model: Theory and evidence

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Abstract. This paper presents an open-economy extension of the Mankiw-Romer-Weil [MRW] model that accounts for technological interdependence among regional economies. Interdependence is assumed to work through spatial externalities caused by disembodied knowledge diffusion. The transition from theory to econometrics leads to a reduced-form empirical model that in the spatial econometrics literature is known as spatial Durbin model specification. We take a system of 198 regions across 22 European countries over the period from 1995 to 2004 to empirically test the model, to analyze the impact of changing the MRW variables on the dependent variable in the model, and – in contrast to previous work – to correctly identify physical and human capital externalities and the strength of technological interdependence among regions.

Keywords: Economic growth, augmented Mankiw-Romer-Weil model, knowledge spillovers, spatial externalities, spatial econometrics, European regions

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1 Introduction

Models of economic growth may be split into two broad categories: neoclassical and endogenous growth models. Neoclassical growth models[1] postulate that physical capital accumulation contributes to the growth in the short-run, but long-run growth is totally determined by technological progress which is exogenous to the models so that there is no explicit role for knowledge and spillovers (Stiroh 2003). In contrast, new growth theory has focused renewed attention on the role of knowledge capital in aggregate economic growth, with a prominent role for knowledge spillovers (see Romer 1986; Grossman and Helpman 1991). Knowledge is inherently non-rival in its use and thus its creation and diffusion most likely leads to spillovers and increasing returns. It is this property of knowledge which is at the centre of endogenous growth models that characteristically treat technological knowledge as completely diffused within an economy[2], and implicitly or explicitly assume that knowledge does not diffuse across economies.

Empirical evidence suggests that technological knowledge spillovers[3] are to a substantial degree geographically localized, in the sense that the productivity effects of knowledge decline with the geographic distance between sender and recipient locations (see Keller 2002, and Fischer et al. 2009). At the same time these studies indicate that there are no good reasons to believe that the flow of technological knowledge stops because it hits national or regional boundaries. The rate at which knowledge diffuses outward from the geographical location in which it is created has important implications for the modelling of technological change and economic growth.

This paper presents an open-economy extension of the Mankiw-Romer-Weil (henceforth MRW) model (see Mankiw et al. 1992), by explicitly accounting for

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[1] Neoclassical growth models are characterized by three central assumptions. *First*, the level of technology is considered as given and, thus, exogenously determined. *Second*, the production function shows constant returns to scale in the production factors for a given, constant level of technology. *Third*, the production factors have diminishing marginal products. This assumption is central to neoclassical growth theory.

[2] There are numerous channels through which knowledge might diffuse. It may be disseminated at conferences, seminars and workshops. It can also be part of the human capital that R&D personnel take with them when changing jobs, or it can be the by-product of mergers and acquisitions, or other forms of interfirm cooperation. It may also be uncovered through reverse engineering and other purposive search processes (Fischer and Varga 2003).

[3] We will use the terms spillovers and externalities in this paper interchangeably, even though they are not synonymous. Knowledge spillovers should be distinguished from rent or pecuniary spillovers that are closely linked to knowledge embodied in traded capital or intermediate goods.

technological interdependence among the economies. Interdependence is assumed to work through spatial externalities caused by disembodied knowledge diffusion. Our research draws on some earlier contributions in different ways. In particular, it models technological progress along the lines suggested by Ertur and Koch (2007), but differs from this work in a number of important directions. *First*, the focus is on an MRW rather than a Solow world of economies in which output is produced from physical capital, human capital and consumption. Accounting for human capital changes one's view of the nature of growth processes. *Second*, the study shifts attention from countries to regions as a more appropriate arena for analyzing growth processes. *Finally*, we empirically test the implied reduced-form empirical model and analyze the direct and indirect impact of changing the MRW variables on the dependent variable, in terms of LeSage and Pace (2009) approach.

With López-Bazo et al. (2004) we share the ambition to extend the MRW model by incorporating spatial externalities, but depart from this study in two major respects. Our focus is on levels rather than on rates of growth. This focus is important because – as Hall and Jones (1999) point out – levels capture the differences in long-run performance which are more directly relevant to welfare as measured by consumption of goods and services. Second, in the light of the recurring criticism in the literature that theoretical models are only loosely connected with empirical evidence (see Levine and Renelt 1991; Durlauf 2001), our study attempts to provide a more explicit and closer link between theory and empirical testing, in an analytical rather than a discursive manner[4].

The remainder of the paper consists of four sections. Section 2 presents the spatially augmented MRW model that accounts for technological interdependence among the regional economies. This transition from theory to econometrics leads to a reduced-form empirical model that in the spatial econometrics literature is known as spatial Durbin model specification. Section 3 describes the relevant methodology for estimating and correctly interpreting the model. Section 4 describes the sample data and summarizes the estimation results. We use a system of 198 regions across 22 European countries over the period from 1995 to 2004, to investigate whether the data support the predictions suggested by the spatially augmented MRW model. Particular emphasis is laid on analyzing the impact of changing the MRW variables on the dependent variable in the model. We use LeSage and Pace's (2009) computational approach to calculating scalar summary measures of these impacts that are decomposed into direct and indirect or spatial externalities. Section 5 offers some closing comments.

[4] In the study by López-Bazo et al. (2004) the predictions of their spatially augmented MRW model are only partially empirically tested, in the sense that the MRW determinants are left out of consideration in the testing exercise.

2 The theoretical model

Consider a world of N regional economies, indexed by i=1, …, N. These economies are similar in that they have the same production possibilities. They differ because of different endowments and allocations. Within a regional economy, all agents are identical. The economies evolve independently in all respects except for technological interdependence.

2.1 The production function and knowledge externalities

Each regional economy is characterized by a (Hicks-neutral) Cobb-Douglas production function, exhibiting constant returns to scale

$$Y_{it} = A_{it}\ K_{it}^{\alpha_K}\ H_{it}^{\alpha_H}\ L_{it}^{1-\alpha_K-\alpha_H} \tag{1}$$

where i denotes the economy and t the time period. Y is output, K the level of reproducible physical capital, H the level of reproducible human capital, L the level of raw labour and A the level of technological knowledge. Moreover, we assume that the same production function applies to physical capital, human capital, and consumption, so that one unit of consumption can be transformed costlessly into either one unit of human capital or one unit of physical capital. The exponents α_K and α_H represent the output elasticities with respect to physical and human capital, respectively. As in Mankiw et al. (1992) we assume $\alpha_K,\ \alpha_H > 0$ and $\alpha_K + \alpha_H < 1$ which implies that there are decreasing returns to both types of capital.

All variables are supposed to evolve in continuous time. The level of labour in economy i grows at rate n_i. Each economy augments its physical and human capital stocks at constant investment rates, s_i^K and s_i^H respectively, while both stocks depreciate at the same rate δ. This induces capital accumulation equations of the form

$$\dot{K}_{it} = s_i^K\ Y_{it} - \delta K_{it} \tag{2a}$$

$$\dot{H}_{it} = s_i^H\ Y_{it} - \delta H_{it} \tag{2b}$$

where the dots over K_{it} and H_{it} represent the derivatives with respect to time. According to Eqs. (2a)-(2b), the change in the capital stocks of region i, $\dot{K}_{it}$ and

$\dot{H}_{it}$, is equal to the amount of gross investment, $s_i^K Y_{it}$ and $s_i^H Y_{it}$ respectively, less the amount of depreciation that occurs during the production process.

The final factor in the production of output is the level of technological knowledge available in region *i* at time *t*. Inspired by Ertur and Koch (2007) we model A_{it} as

$$A_{it} = \Omega_t \; k_{it}^{\theta} \; h_{it}^{\phi} \prod_{j \neq i}^{N} A_{jt}^{\rho W_{ij}} . \tag{3}$$

Several aspects of modelling the aggregate level of technology deserve mentioning. *First*, the term Ω_t is used – as in Mankiw et al. (1992) – to represent that the amount of knowledge created anywhere in the world of regions which is immediately available to be used in any economy. This part of the knowledge stock is identical in all regions and exogenous to the model: $\Omega_t = \Omega_0 \exp(\mu t)$ where μ is its constant rate of growth.

Second, we assume that technology is embodied in physical and human capital per worker and that region's *i* aggregate level of technology increases with both the aggregate level of physical capital per worker, $k_{it} = K_{it} / L_{it}$, and the aggregate level of human capital per worker, $h_{it} = H_{it} / L_{it}$. The associated parameters θ with $0 \leq \theta < 1$ and ϕ with $0 \leq \phi < 1$ reflect spatial connectivity of k_{it} and h_{it} within region *i*, respectively[5].

Finally, we assume non-embodied knowledge to cause the technological progress of region *i* to positively depend on the technological progress of other regions $j \neq i$, for $j = 1, ..., N$. The last term on the right hand side of Eq. (3) formalizes the spatial extent of this dependence by means of so-called spatial weight terms W_{ij} that represent the spatial connectivity between regions *i* and *j*, for $j = 1,..., N$. These terms are assumed to be non-negative, non-stochastic and finite, with the properties $0 \leq W_{ij} \leq 1$, $W_{ij} = 0$ if $i = j$, and $\sum_{j=1}^{N} W_{ij} = 1$ for $i = 1,..., N$. The parameter ρ with $0 \leq \rho < 1$ reflects the degree of regional interdependence[6]. Regions neighbouring region *i* are defined as those regions *j* for which $W_{ij} > 0$. The more a region *i* is connected to region *j*, the higher W_{ij} is, and the more region *i* benefits from knowledge spilling over from region *j*.

Rewriting the log-version of Eq. (3) in matrix form at time *t* yields

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[5] We assume hereby that each unit of capital investment increases not only the stock of capital, but also generates externalities which lead to knowledge spillovers that increase the level of technology for all firms in the region.

[6] Even though ρ is a global parameter characterizing the degree of technological interdependence in the system of regions, the net effect of this dependence on the productivity level of the firms in region *i* depends on the spatial connectivity relationship incorporated in the model (see LeSage and Fischer 2008).

$$\boldsymbol{A} = \boldsymbol{\Omega} + \theta \boldsymbol{k} + \phi \boldsymbol{h} + \rho \boldsymbol{W} \boldsymbol{A} \tag{4}$$

where $\boldsymbol{A}$ is the (N, 1) vector of the level of knowledge for the N regions, $\boldsymbol{\Omega}$ is the (N, 1) vector of the exogenous part of technology, $\boldsymbol{k}$ and $\boldsymbol{h}$ are the (N, 1) vectors of per worker physical and human capital respectively. $\boldsymbol{W}$ denotes the (N, N) matrix of spatial weights representing the spatial connectivity structure between the N regions. If $\rho \neq 0$ and if ρ^{-1} is not an eigenvalue of $\boldsymbol{W}$, we can resolve Eq. (4) for $\boldsymbol{A}$, yielding

$$\boldsymbol{A} = (\boldsymbol{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{\Omega} + \theta (\boldsymbol{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{k} + \phi (\boldsymbol{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{h} \,. \tag{5}$$

Using the Sherman-Morrison formula to develop $(\boldsymbol{I} - \rho \boldsymbol{W})^{-1}$ in its Taylor expansion form and regrouping terms, we get[7] for a region i

$$A_{it} = \Omega_t^{\frac{1}{1-\rho}} \; k_{it}^{\theta} \; h_{it}^{\phi} \prod_{j \neq i}^{N} k_{jt}^{\theta \sum_{r=1}^{\infty} \rho^r (\boldsymbol{W}^r)_{ij}} \; h_{jt}^{\phi \sum_{r=1}^{\infty} \rho^r (\boldsymbol{W}^r)_{ij}} \tag{6}$$

where $(\boldsymbol{W}^r)_{ij}$ denotes the (i, j)th element of $\boldsymbol{W}^r$. Inserting this equation in the per worker production function, given by normalizing Eq. (1) by L_{it}, we obtain the theoretical model[8]

$$y_{it} = \Omega_t^{\frac{1}{1-\rho}} \; k_{it}^{u_{ii}} \; h_{it}^{v_{ii}} \prod_{j \neq i}^{N} k_{jt}^{u_{ij}} \; h_{jt}^{v_{ij}} \tag{7a}$$

$$u_{ii} = \alpha_K + \theta \left(1 + \sum_{r=1}^{\infty} \rho^r \, (\boldsymbol{W}^r)_{ii} \right) \tag{7b}$$

$$u_{ij} = \theta \sum_{r=1}^{\infty} \rho^r \, (\boldsymbol{W}^r)_{ij} \quad \text{for } i \neq j \tag{7c}$$

7 Note that $(\boldsymbol{I} - \rho \boldsymbol{W})^{-1} = \Sigma_{r=0}^{\infty} (\rho \boldsymbol{W})^r = \Sigma_{r=0}^{\infty} \rho^r (\boldsymbol{W}^r)$, $\Sigma_{r=0}^{\infty} \boldsymbol{W}^r$ is row standardized since $\boldsymbol{W}$ is so, $\Sigma_{r=0}^{\infty} \boldsymbol{W}^r \boldsymbol{\Omega} = \boldsymbol{\Omega}$, $\Sigma_{r=0}^{\infty} \rho^r = 1/(1-\rho)$ if $|\rho| < 1$.

8 Note that this model would become an endogenous growth model if $\alpha_K + \alpha_H = 1$. Then there are constant returns to scale in the reproducible factors. In this case, there is no steady state for the model. Regions that invest more would grow faster indefinitely.

$$v_{ii} = \alpha_H + \phi \left(1 + \sum_{r=1}^{\infty} \rho^r (\mathbf{W}^r)_{ii} \right) \tag{7d}$$

$$v_{ij} = \phi \sum_{r=1}^{\infty} \rho^r (\mathbf{W}^r)_{ij} \quad \text{for } i \neq j. \tag{7e}$$

Equation (7a) relates per worker output, $y_{it} = Y_{it} / L_{it}$, in region i to physical and human capital intensities in the same region and its neighbours j, with $j \neq i$. Note that if $\theta = \phi = 0$, then the model collapses to the MRW model with $Y_{it} = \Omega_t \, K_{it}^{\alpha_K} \, H_{it}^{\alpha_H} \, L_{it}^{1-\alpha_K-\alpha_H}$, which is characterized by a world of closed economies.

We can evaluate the social elasticity of output per worker in region i at time t with respect to both types of capital per worker. From Eqs. (7a) to (7e) it is evident that when region i increases its own stocks of per worker physical and human capital, it receives a social return of

$$\frac{\partial y_{it}}{\partial k_{it}} \frac{k_{it}}{y_{it}} + \frac{\partial y_{it}}{\partial h_{it}} \frac{h_{it}}{y_{it}} = u_{ii} + v_{ii} \tag{8a}$$

whereas this return increases to

$$\begin{aligned} &\frac{\partial y_{it}}{\partial k_{it}} \frac{k_{it}}{y_{it}} + \sum_{j \neq i}^{N} \frac{\partial y_{it}}{\partial k_{jt}} \frac{k_{jt}}{y_{it}} + \frac{\partial y_{it}}{\partial h_{it}} \frac{h_{it}}{y_{it}} + \sum_{j \neq i}^{N} \frac{\partial y_{it}}{\partial h_{jt}} \frac{h_{jt}}{y_{it}} = \\ &\qquad = u_{ii} + \sum_{j \neq i}^{N} u_{ij} + v_{ii} + \sum_{j \neq i}^{N} v_{ij} \end{aligned} \tag{8b}$$

if all regions simultaneously increase their per worker stocks as well.

2.2 Transitional dynamics and the steady state

The evolution of output per worker in region i is governed by the dynamic equations for k and h given by

$$\dot{k}_{it} = s_i^K y_{it} - (n_i + \delta) \, k_{it} \tag{9a}$$

$$\dot{h}_{it} = s_i^H \, y_{it} - (n_i + \delta) \, h_{it} \tag{9b}$$

where s_i^K is the fraction of output in region i invested in physical capital, s_i^H the fraction of output invested in human capital, n_i the rate of population growth and δ the constant and identical rate of depreciation.

Since the per worker production function given by Eq. (7a) is characterized by decreasing returns to both types of capital, Eqs. (9a) and (9b) imply that per worker output of region i, for i=1, …, N, converges to a steady state[9] defined by

$$y_{it}^* = \Omega_t^{\frac{1}{(1-\rho)(1-u_{ii}-v_{ii})}} \left[\frac{(s_i^K)^{u_{ii}} \, (s_i^H)^{v_{ii}}}{(n_i + g + \delta)^{u_{ii}+v_{ii}}} \right]^{\frac{1}{1-u_{ii}-v_{ii}}} \prod_{j \neq i}^{N} \left(k_{jt}^{*\,u_{ij}} \;\; h_{jt}^{*\,v_{ij}} \right)^{\frac{1}{1-u_{ii}-v_{ii}}} \tag{10a}$$

with the balanced growth rate[10]

$$g = \frac{\mu}{(1-\rho)(1-\alpha_K - \alpha_H) - \theta - \phi} \tag{10b}$$

where the asterisk is used to signify the steady state levels for y, k and h. Hence, the physical capital-output and human capital-output ratios of region i, for i=1, …, N, are constant so that

$$\frac{k_{it}^*}{y_{it}^*} = \frac{s_i^K}{n_i + g + \delta} \tag{11a}$$

$$\frac{h_{it}^*}{y_{it}^*} = \frac{s_i^H}{n_i + g + \delta}. \tag{11b}$$

[9] Note that the balanced growth path is defined as a situation in which (i) per worker physical and human capital grow at the same rate denoted by g, (ii) the exogenous part of technology grows at the constant rate μ, and (iii) the population growth rate and the investment rates for physical and human capital are constant.

[10] This follows from solving $\partial \ln y_{it} / \partial t = \dot{\Omega}_t / \Omega_t + \rho \, \Sigma_{j \neq i} \, (\dot{y}_{jt} / y_{jt}) \; + (\alpha_K + \theta) \; (\dot{k}_{it} / k_{it}) +$ $+ (\alpha_H + \phi) \; (\dot{h}_{it} / h_{it}) - \alpha_K \, \rho \Sigma_{j \neq i} W_{ij} \; (\dot{k}_{jt} / k_{jt}) - \alpha_H \, \rho \Sigma_{j \neq i} W_{ij} \; (\dot{h}_{jt} / h_{jt})$ for g at the balanced growth path.

Substituting these expressions of capital-output ratios at steady state into the per worker production function and taking the logarithm, gives an equation to the output per worker of region *i* at steady state:

$$\ln y_{it}^{*} = \frac{1}{1-\eta}\ln \Omega_t + \frac{\alpha_K+\theta}{1-\eta}\ln s_i^K + \frac{\alpha_H+\phi}{1-\eta}\ln s_i^H - \frac{\eta}{1-\eta}\ln(n_i+g+\delta)$$

$$-\frac{\alpha_K}{1-\eta}\rho\sum_{j\neq i}^{N}W_{ij}\ln s_j^K - \frac{\alpha_H}{1-\eta}\rho\sum_{j\neq i}^{N}W_{ij}\ln s_j^H +$$

$$+\frac{\alpha_K+\alpha_H}{1-\eta}\rho\sum_{j\neq i}^{N}W_{ij}\ln(n_j+g+\delta) + \frac{1-\alpha_K-\alpha_H}{1-\eta}\rho\sum_{j\neq i}^{N}W_{ij}\ln y_{jt}^{*} \qquad (12)$$

with $\eta=\alpha_K+\alpha_H+\theta+\phi$. This equation shows how output per worker at steady state depends on population growth and accumulation of physical and human capital not only in the region itself, but also in neighbouring regions. It is important to note that Eq. (12) is valid only if the regions are in their steady states or, more generally, deviations from steady state are random.

This spatially augmented neoclassical growth model has the same qualitative predictions as the MRW model. The per worker output of region *i* at steady state depends positively on its own physical capital and human capital investment rates ($\ln s_i^K$ and $\ln s_i^H$) and negatively on its population growth rate $\ln(n_i+g+\delta)$. Per worker output of region *i*, however, depends also on determinants that lie outside MRW's original theory. The per worker output of a region *i* at steady state is negatively influenced by investment rates for physical and human capital investment rates in neighbouring regions j, for $j\neq i$, those identified by $W_{ij}>0$, and positively influenced by their population growth rates. Output (per worker) of region *i* also depends on (per worker) steady state levels in neighbouring regions. These output levels ($\ln y_j^{*}$) of neighbouring regions in turn depend on the MRW variables, so that changes in explanatory variables will affect the dependent variable $\ln y_i^{*}$. We note that if $\theta=\phi=\rho=0$, Eq. (12) reduces to the conventional MRW steady state equation[11].

[11] It is interesting to note that our spatially augmented MRW steady state equation collapses to the Ertur-Koch model (Ertur and Koch 2007), if and only if human capital does not play a significant role in regional growth processes and, thus, $\alpha_H=\phi=0$.

3 Model specification, estimation and interpretation

This section describes the transition from the reduced-form of our theoretical model (12) to the spatial econometric model specification used in this study. Section 3.1 presents the empirical model associated with the reduced-form of the theoretical model, Section 3.2 outlines the approach for parameter estimation. Section 3.3 directs attention to correctly interpret the parameter estimates.

3.1 The spatial econometric specification

It is easy to see that the empirical counterpart of model (12) can be expressed at a given time (*t*=0 for simplicity) in the following form for region *i*

$$\ln y_i = \beta_0 + \beta_1 \ln s_i^K + \beta_2 \ln s_i^H + \beta_3 \ln (n_i + g + \delta) + \gamma_1 \sum_{j \neq i}^{N} W_{ij} \ln s_j^K$$
$$+ \gamma_2 \sum_{j \neq i}^{N} W_{ij} \ln s_j^H + \gamma_3 \sum_{j \neq i}^{N} W_{ij} \ln (n_j + g + \delta) + \lambda \sum_{j \neq i}^{N} W_{ij}\, y_j + \varepsilon_i \qquad (13)$$

where $(1-\eta)^{-1} \ln \Omega_0 = \beta_0 + \varepsilon_i$ for $i = 1, ..., N$, with β_0 a constant and ε_i a region-specific shift or shock term[12]. Note that we have the following theoretical constraints between coefficients: $\beta_1 + \beta_2 + \beta_3 = 0$ and $\gamma_1 + \gamma_2 + \gamma_3 = 0$, since the theoretical model predicts not only the signs, but also the magnitudes of the coefficients on the MRW variables and their spatial lags.

Rewriting Eq. (13) in matrix form yields

$$\boldsymbol{y} = \boldsymbol{\iota}_N\, \beta_0 + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{W}\,\boldsymbol{X}\,\boldsymbol{\gamma} + \lambda \boldsymbol{W}\,\boldsymbol{y} + \boldsymbol{\varepsilon} \qquad (14)$$

where

- $\boldsymbol{y}$ — *N*-by-1 vector of observations on the per worker output level for each of the *N* regions,
- $\boldsymbol{X}$ — *N*-by-*Q* matrix of observations on the *Q* non-constant exogenous variables [here *Q*=3], including the vectors of the physical and human capital investment rates and the population growth rate for each of the *N* regions,

[12] The term Ω_0 reflects – as Mankiw et al. (1992) emphasize – not just technology, but also resource endowments, institutions and so on, and hence may vary across the regions.

$\boldsymbol{\beta}$	Q-by-1 vector of the regression parameters associated with the Q non-constant exogenous variables [here: $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)'$],
$\boldsymbol{WX}$	N-by-Q matrix of the Q spatially lagged non-constant exogenous variables,
$\boldsymbol{\gamma}$	Q-by-1 vector of the regression parameters associated with the Q spatially lagged non-constant exogenous variables [here: $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \gamma_3)'$],
$\boldsymbol{W y}$	N-by-1 vector of the dependent spatial lag variable that contains a linear combination of the per worker output levels from neighbouring regions, those identified by $W_{ij} > 0$,
λ	the spatial autocorrelation coefficient, where $\lambda = (1 - \alpha_K - \alpha_H)\rho / (\eta - 1)$,
$\boldsymbol{\iota}_N$	N-by-1 vector of ones with the associated scalar parameter β_0,
$\boldsymbol{\varepsilon}$	N-by-1 vector of errors assumed to be identically and normally distributed with zero mean: $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \boldsymbol{I})$.

All variables are in log form. The variables spanned by $\boldsymbol{X}$ represent the determinants that are suggested by the MRW model, whereas $\boldsymbol{WX}$ represent those that lie outside MRW's original theory, as does $\boldsymbol{W y}$ that represents the technological interdependence between the regions and defines the difference to a MRW world of closed regions.

In the spatial econometrics literature, a model specification like Eq. (14) that includes spatial lags of the dependent and independent variables is referred to as a spatial Durbin model. For ease of exposition, we rewrite Eq. (14) as

$$\boldsymbol{y} = \boldsymbol{U}\,\boldsymbol{\varphi} + \lambda \boldsymbol{W y} + \boldsymbol{\varepsilon} \tag{15}$$

with $\boldsymbol{U} = [\boldsymbol{\iota}_N\ \boldsymbol{X}\ \boldsymbol{WX}]$ and $\boldsymbol{\varphi} = [\beta_0\ \boldsymbol{\beta}\ \boldsymbol{\gamma}]'$. If $\lambda \neq 0$ and if λ^{-1} is not an eigenvalue of $\boldsymbol{W}$, $(\boldsymbol{I} - \lambda \boldsymbol{W})$ is non-singular, and we get model specification (15) in reduced form as

$$\boldsymbol{y} = (\boldsymbol{I} - \lambda \boldsymbol{W})^{-1} \boldsymbol{U}\,\boldsymbol{\varphi} + (\boldsymbol{I} - \lambda \boldsymbol{W})^{-1} \boldsymbol{\varepsilon}. \tag{16}$$

From this equation it follows that the spatially lagged variable $\boldsymbol{Wy}$ is correlated since

$$E(\boldsymbol{Wy}\ \boldsymbol{\varepsilon}) = \sigma^2 \boldsymbol{W}\,(\boldsymbol{I} - \lambda \boldsymbol{W})^{-1} \neq 0. \tag{17}$$

Thus, OLS estimators will be biased and inconsistent. Instrumental variables, Bayesian and maximum likelihood (ML) based estimators provide consistent estimates of a model such as that one given by Eq. (15) or its reduced form (16).

3.2 The log likelihood function and ML estimation

In this study we apply the maximum likelihood approach. Given $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \boldsymbol{I})$, the log likelihood function for $\boldsymbol{\varphi}$, λ and the noise variance parameter σ^2 is

$$\ln \mathcal{L}(\boldsymbol{\varphi}, \lambda, \sigma^2) = -\frac{N}{2}(\ln 2\pi + \ln \sigma^2) + \ln | \boldsymbol{I} - \lambda \boldsymbol{W} | - \frac{1}{2\sigma^2} [(\boldsymbol{I} - \lambda \boldsymbol{W})\boldsymbol{y} - \boldsymbol{U}\boldsymbol{\varphi}]' [(\boldsymbol{I} - \lambda \boldsymbol{W})\boldsymbol{y} - \boldsymbol{U}\boldsymbol{\varphi}]. \tag{18}$$

From the usual first-order conditions, the ML estimates of $\boldsymbol{\varphi}$ and σ^2, given λ, are obtained as

$$\hat{\boldsymbol{\varphi}}(\lambda) = (\boldsymbol{U}'\boldsymbol{U})^{-1} \boldsymbol{U}' (\boldsymbol{I} - \lambda \boldsymbol{W})\ \boldsymbol{y} \tag{19}$$

$$\hat{\sigma}^2(\lambda) = \frac{1}{N} [(\boldsymbol{I} - \lambda \boldsymbol{W})\boldsymbol{y} - \boldsymbol{U}\ \hat{\boldsymbol{\varphi}}(\lambda)]' [(\boldsymbol{I} - \lambda \boldsymbol{W})\boldsymbol{y} - \boldsymbol{U}\ \hat{\boldsymbol{\varphi}}(\lambda)]. \tag{20}$$

Note that $\hat{\boldsymbol{\varphi}}(\lambda) = \hat{\boldsymbol{\varphi}}_0 - \lambda \hat{\boldsymbol{\varphi}}_L$ with $\hat{\boldsymbol{\varphi}}_0 = (\boldsymbol{U}'\boldsymbol{U})^{-1} \boldsymbol{U}'\boldsymbol{y}$ and $\hat{\boldsymbol{\varphi}}_L = (\boldsymbol{U}'\boldsymbol{U})^{-1} \boldsymbol{U}'\boldsymbol{W}\ \boldsymbol{y}$. Define $\hat{\boldsymbol{e}}_0 = \boldsymbol{y} - \boldsymbol{U}\ \hat{\boldsymbol{\varphi}}_0$ and $\hat{\boldsymbol{e}}_L = \boldsymbol{W}\ \boldsymbol{y} - \boldsymbol{U}\ \hat{\boldsymbol{\varphi}}_L$ then it is easy to see that $\hat{\sigma}^2 = \frac{1}{N}(\hat{\boldsymbol{e}}_0 - \lambda \hat{\boldsymbol{e}}_L)' (\hat{\boldsymbol{e}}_0 - \lambda \hat{\boldsymbol{e}}_L)$. Substituting Eqs. (19)-(20) into Eq. (18) gives the concentrated log likelihood that depends only on the single parameter λ:

$$\ln \mathcal{L}(\lambda) = \frac{N}{2} \ln 2\pi + \ln | \boldsymbol{I} - \lambda \boldsymbol{W} | - \frac{1}{2} \ln (\hat{\boldsymbol{e}}_0 - \lambda \hat{\boldsymbol{e}}_L)' (\hat{\boldsymbol{e}}_0 - \lambda \hat{\boldsymbol{e}}_L) \tag{21}$$

where $\hat{\boldsymbol{e}}_0$ and $\hat{\boldsymbol{e}}_L$ are the estimated residuals in a regression of $\boldsymbol{y}$ on $\boldsymbol{U}$ and $\boldsymbol{Wy}$ on $\boldsymbol{U}$, respectively.

Working with this likelihood function concentrated with respect to the parameters $\boldsymbol{\varphi}$ and σ^2 yields exactly the same ML estimates $\hat{\boldsymbol{\varphi}}$, $\hat{\sigma}$ and $\hat{\lambda}$ as would arise from maximizing the full log likelihood function given in Eq. (18) (see Davidson and MacKinnon 1993, pp. 267-269). A variety of univariate optimization techniques may be used for optimizing the concentrated log likelihood function. In this study we use the simplex optimization algorithm.

The main difficulty in numerical maximization is the necessity of evaluating the log-determinant of the N-by-N matrix $(\boldsymbol{I}-\lambda\boldsymbol{W})$ afresh at each iteration step of the optimization process. To minimize the computational burden, Ord (1975) suggested to express the troublesome term, $\ln|\boldsymbol{I}-\lambda\boldsymbol{W}|$, in Eq. (21) as

$$\ln|\boldsymbol{I}-\lambda\boldsymbol{W}|=\sum_{i=1}^{N}\ln(1-\lambda\,\kappa_i) \tag{22}$$

where κ_i denotes the ith eigenvalue of $\boldsymbol{W}$. The advantage of this so-called eigenvalue route to compute the log-determinant is that $\{\kappa_i | i=1,...,N\}$ can be determined once at the outset of the numerical optimization process.

3.3 Interpreting the parameter estimates

The empirical model (13) associated with the reduced-form of the theoretical model in Eq. (12) provides very rich own- and cross-partial derivatives that quantify the magnitude of direct and indirect or spatial spillover effects which arise from changes in region i's characteristics (human capital accumulation, physical capital accumulation and population growth). This arises from introducing technological interdependence among a few neighbours at the outset in the theoretical model. A logical consequence of the simple dependence on a small number of nearby regions in the theoretical specification of the aggregate level of technology leads to a final-form model outcome where changes in a single region can potentially impact all other regions. Of course, there is a decay of influence, as one moves to more distant regional economies.

The inherent complexity of the spatial Durbin model specification given by Eq. (13) – or by Eq. (14) in matrix form – means that treating the parameter estimates like least-squares parameter estimates is incorrect, as noted by LeSage and Fischer (2008), and LeSage and Pace (2009). A change in a single observation of an explanatory variable will affect the regional economy itself (a *direct impact*) and potentially affect all other economies indirectly (an *indirect impact* or *spatial spillover effect*). Direct and indirect effects correspond to the own- and cross-partial derivatives of $\ln y_i$ with respect to the ith and jth observation of X_q (the qth MRW variable, q=1, …, Q=3) which we denote by X_{iq} and X_{jq} respectively so that

$$\frac{\partial \ln y_i}{\partial X_{iq}} \neq \beta_q \qquad \text{for } i=1,...,N;\ q=1,...,Q \tag{23a}$$

$$\frac{\partial \ln y_i}{\partial X_{jq}} \neq 0 \qquad \text{for } j \neq i;\ i, j = 1,...,N;\ q = 1,...,Q. \tag{23b}$$

Our empirical model with the Q=3 MRW variables leads to QN^2 partial derivatives to examine, and this provides too much information to easily digest.

To quantify these complex spatial interactions we rely on LeSage and Pace's (2009) approaches to calculating scalar summary measures of the direct and indirect impacts of the three MRW variables. The direct impact is summarized using the average impact of a change in the explanatory variable at each of the N locations on the dependent variable at the same location. The indirect impact – reflecting spatial spillovers – is summarized by the average impact of a change in the explanatory variable at each location on the dependent variable at different locations.

Formally, LeSage and Pace (2009, pp. 36-37) define these summary impact measures as follows:

(i) *The average direct impact*. The impact of changes in the ith observation of X_q on $\ln y_i$ can be summarized by measuring the average $\boldsymbol{S}_q(\boldsymbol{W})_{ii}$, which equals $N^{-1}tr(\boldsymbol{S}_q(\boldsymbol{W}))$ where $\boldsymbol{S}_q(\boldsymbol{W})_{ii}$ is the (i, i)th element of the N-by-N matrix

$$\boldsymbol{S}_q(\boldsymbol{W}) = (\boldsymbol{I} - \lambda \boldsymbol{W})^{-1} (\boldsymbol{I} \beta_q + \boldsymbol{W} \gamma_q) \tag{24}$$

for $q = 1, ..., Q$. The diagonal elements of $\boldsymbol{S}_q(\boldsymbol{W})$ contain the direct impacts so that the average direct effect is constructed as an average of the diagonal elements.

Note: $\boldsymbol{S}_q(\boldsymbol{W})_{ii}$ measures the impact on the dependent variable observation i, $\ln y_i$, from a change in X_{iq}. This impact includes the effect of feedback loops, where observation i affects observation j and observation j also affects observation i. The magnitude of this type of feedback depends on *first*, the location of the regions in geographic space; *second*, the degree of connectivity among the regions governed by the spatial weight matrix $\boldsymbol{W}$; *third*, the parameter λ that measures the strength of technological interdependence among the regions; and *finally*, the parameters $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

(ii) *The average indirect impact*. The indirect effects that arise from changes in all observations $j = 1,...,N$ of an explanatory variable are found as the sum of the off-diagonal elements of row i from the matrix $\boldsymbol{S}_q(\boldsymbol{W})$ given by Eq. (24). The average indirect impact is constructed as an average of the off-diagonal elements, where the off-diagonal row elements are summed up first, and then an average of these sums is taken.

Computing these direct and indirect summary impacts requires little additional computational cost. The low cost of computation allows simulating the distribution of the impacts to derive inference statistics based on the maximum likelihood parameter estimates.

4 Testing the spatially augmented MRW model

In this section we consider the question whether data for European regions support the predictions suggested by the spatially augmented MRW model. Using the empirical model in Eq. (14), we estimate the direct and indirect effects of the three MRW variables, and assess the role played by regional technological interdependence in the growth process.

4.1 Sample data and the spatial weight matrix

Our sample is a cross-section of 198 regions belonging to 22 European countries over the 1995-2004 period. The units of observation are the NUTS-2 regions. These regions, though varying in size, are generally considered to be the most appropriate spatial units for modelling and analysis purposes (Fingleton 2001). In most cases, they are sufficiently small to capture subnational variations. But we are aware that NUTS-2 regions are formal rather than functional regions, and their delineation does not represent the boundaries of regional growth processes very well. The choice of the NUTS-2 level might also give rise to a form of the modifiable areal unit problem, well known in geography (see, for example, Getis 2005).

The sample regions include regions located in *Western Europe* as well as in *Eastern Europe*. Western Europe is represented by 159 regions[13] covering Austria (nine regions), Belgium (11 regions), Denmark (one region), Finland (four regions), France (21 regions), Germany (40 regions), Italy (18 regions), Luxembourg (one region), the Netherlands (12 regions), Norway (seven regions), Portugal (five regions), Spain (15 regions), Sweden (eight regions) and Switzerland (seven regions). *Eastern Europe* is covered by 39 regions including

[13] We exclude the Spanish North African territories of Ceuta y Melilla, the Spanish Balearic islands, the Portuguese non-continental territories Azores and Madeira, the French Départements d'Outre-Mer Guadaloupe, Martinique, French Guayana and Réunion, and, moreover, Åland (Finland), Corse, Sardegna and Sicilia.

the Baltic states (three regions), the Czech Republic (eight regions), Hungary (seven regions), Poland (16 regions), Slovakia (four regions) and Slovenia (one region). The main data source is Eurostat's Regio database[14]. The data for Norway and Switzerland were provided by Statistics Norway and the Swiss Office Fédéral de la Statistique, respectively.

The data cover the period from 1995 to 2004 when economic recovery in Eastern Europe gathered pace. The time period is relatively short[15] due to a lack of reliable figures for the regions in Eastern Europe (Fischer and Stirböck 2006). The political changes since 1989 have resulted in the emergence of new or re-established states (the Baltic states, the Czech Republic, Slovakia and Slovenia) with only a very short history as sovereign national entities. In most of these states historical data series simply do not exist. Even for states such as Hungary and Poland that existed for much longer time periods in their present boundaries, the quality of data referring to the period of central planning imposes serious limitations on analyzing regional growth. This is closely related to the change in accounting conventions, from the material product balance system to the European System of Accounts 1995. Cross-region comparisons require internationally comparable regional data which are not only statistically consistent but also expressed in the same numéraire. The absence of market exchange rates in the former centrally planned economies is a further impediment.

We focus on an output based measure and use gross value added, *gva*, rather than gross regional product at market prices as a proxy for regional output. *gva* is the net result of output at basic prices less intermediate consumption valued at purchasers' prices, and measured in accordance with the European System of Accounts 1995. The dependent variable is *gva* divided by the number of workers in 2004[16]. We measure n as the growth rate of the working age population, where working age is defined as 15-64 years, and use gross fixed capital formation per worker as a proxy for physical capital investment. Following Mankiw et al. (1992), we restrict our focus on investment in human capital in the form of education and take a proxy for the rate of human capital accumulation that measures the percentage of the working age population (15 years and older) with higher education as defined by the International Standard Classification of Education (ISCED) 1997 classes five and six. n_i, s_i^K and s_i^H are averages for

[14] The data used for labour stem from the Cambridge Econometrics database.

[15] Islam (1995) and Durlauf and Quah (1999) emphasize growth regressions of the type considered in this paper are also valid for shorter time spans since they are steady state regressions.

[16] To implement the model we have been assuming that the regions were in their steady state in 2004 (or more generally, that the deviations from steady state were random).

the period 1995-2003. We suppose[17] that $g + \delta = 0.05$, which is a fairly standard assumption in the literature (see among others, Mankiw et al. 1992; Islam 1995; Temple 1998; Durlauf and Johnson 1995; Ertur and Koch 2007; Fingleton and Fischer 2009).

The definition of the spatially lagged variables in the model depends on the specification of the spatial weight matrix that summarizes the spatial connectivity structure between the regions. Different spatial weight matrices may be chosen[18]. In this study, we employ a binary first-order contiguity matrix, constructed on the basis of digital boundary files in a GIS and implemented in row-standardized form in order to assign equal weight to all contiguous neighbouring regions. Two regions are defined as neighbors when they share a common boundary. This choice of the spatial weight matrix is well in line with the empirical evidence that knowledge spillovers and their productivity effects are to a substantial degree localized (see Fischer et al. 2009).

4.2 Estimation results

We begin by briefly considering the ML-parameter estimates and associated implied parameter values from our spatially augmented MRW model. Table 1 summarizes these estimates along with some diagnostics and performance measures. Diagnostic tests were carried out for heteroskedasticity, using the spatial Breusch-Pagan test[19], and for normality, using the Jarque-Bera test[20]. Performance of the model is expressed in terms of conventional statistical measures of goodness of fit, such as the log likelihood value divided by N, the noise variance sigma square, and R^* defined as the correlation between the fitted and observed values of the dependent variable.

Table 1 about here

[17] There are no strong reasons to expect g and δ to vary greatly across regions, nor are there any data that would allow us to estimate region-specific balanced growth and depreciation rates.

[18] For extensive reviews see Cliff and Ord (1981), Anselin (1988), Anselin and Bera (1998), and Griffith (1995). The latter provides some guidelines for specifying the weight matrix.

[19] This test points to homogeneity in the unconstrained estimation of the spatially augmented MRW model, but reveals heterogeneity in the constrained estimation.

[20] The Jarque-Bera test indicates a lack of normality. Because of the large sample, the test is very powerful, detecting significant deviations from normality which have, however, little significance in practice.

The first three columns of the table present the results imposing the theoretical constraints $\beta_1 + \beta_2 + \beta_3 = 0$ and $\gamma_1 + \gamma_2 + \gamma_3 = 0$ on estimating the model, the final three columns report those without imposing the constraints. The parameter estimates are given in the first and fourth column, followed by the standard errors in the second and fifth, and the *p*-values in the third and sixth column. The parameters obtained by constrained or unconstrained estimation allow us to calculate output elasticity parameters α_K and α_H, and implied values of the parameters θ, ϕ and ρ. To draw inferences regarding the statistical significance of these parameters we calculated measures of dispersion based on simulating parameters from the normally distributed parameters φ_1, φ_2, φ_3, γ_1, γ_2, γ_3, λ, and $\sigma^2_{\boldsymbol{\varepsilon}}$, using the estimated means and variance-covariance matrix. The simulated draws are then used in computationally efficient formulas to calculate the implied distribution of the output elasticity and externality parameters.

Five aspects of the results presented in the table support our spatially augmented MRW model. *First*, the coefficients of all the determinants have the predicted signs and are highly significant. The only exception is the γ_3 parameter estimate for population growth that has the correct sign, but is insignificant. *Second*, the estimates of the output elasticities implied by the raw parameter estimates are empirically plausible. The elasticity of output with respect to the stock of physical capital is very close to one-third. The implied value of α_H is significant, but smaller by a factor of about two. It is interesting to recognize that the implied parameter values obtained from constrained and unconstrained estimation are strikingly similar.

Third, the spatial autocorrelation λ is positive and significant. The implied value of ρ that measures the degree of technological interdependence among the regions is 0.63 with a standard deviation of 0.07 (p=0.00) in the case of unconstrained estimation, and 0.74 with a standard deviation of 0.05 (p=0.00) in the constrained case.

Fourth, a common factor test using likelihood ratios (see LeSage and Pace 2009 for details) rejects the three non-linear restrictions[21]: $\gamma_1 + \beta_1 \rho = 0$, $\gamma_2 + \beta_2 \rho = 0$ and $\gamma_3 + \beta_3 \rho = 0$. The likelihood is 225.61 for the spatially augmented model specification and 192.69 for the MRW model with spatial error terms, based on the binary first-order contiguity matrix and non-constrained estimation. This leads to a difference of 32.92, and this represents a rejection of the spatial error model in favour of the spatial Durbin model specification using the 99 percent critical value for which $\chi^2(3)$ equals 11.34. This result is in

[21] The model specification with these restrictions is then the so-called constrained SDM, which is formally equivalent to a MRW model with spatial autoregressive errors that may be written in matrix form as $\boldsymbol{y} = \boldsymbol{X}\,\boldsymbol{\gamma} + \boldsymbol{\varepsilon}_{MRW}$ with $\boldsymbol{\varepsilon}_{MRW} = \rho \boldsymbol{W}\,\boldsymbol{\varepsilon}_{MRW} + \boldsymbol{\varepsilon}$ where $\boldsymbol{\varepsilon}_{MRW}$ is the same as before if $\theta = 0 = \phi$.

accordance with López-Bazo and Fingleton (2006), questioning the credibility of specifications with dependence structures in the error terms.

Finally, differences in the MRW variables and their spatial lags account for a large fraction of the cross-region variation in per worker output. The measure R^* of the overall fit of the model, defined as the correlation between the fitted and observed values of the dependent variable, ranges from 0.949 (constrained estimation) to 0.966 (unconstrained estimation). Nonetheless, the model is not completely successful, since the joint theoretical restrictions between the parameters are rejected by a likelihood ratio test.

As indicated in Section 3.3, we need to interpret the magnitude of the coefficient estimates from spatial regression models in light of the spatial dependence structure and, hence, need to analyze the impact of changing the MRW variables on the dependent variable in the model. Table 2 presents the estimates for direct and indirect effects of the MRW variables, along with their associated statistics. A comparison of the direct impact and the raw parameter estimates presented in the table for reference shows that these two sets of estimates are not so dissimilar in magnitude. The direct impact of the human capital variable is slightly lower, while that of the physical capital variable is somewhat larger than one would infer from the coefficient estimates. The difference between these estimates is due to feedback effects.

Table 2 about here

Turning to the indirect impact estimates in Table 2, we note larger differences between these estimates and the coefficients on the spatially lagged explanatory variables. The estimates associated with the spatially lagged variables are often interpreted (incorrectly) as measures of the size and significance of indirect impacts in spatial regression models. As the discrepancies in the table indicate, this could lead to incorrect inferences about the true role of neighbouring regions' characteristics. For example, the coefficient reported for human capital accumulation is -0.14 with a standard deviation of 0.04 and highly significant (p=0.00), whereas the mean indirect impact for this variable is not significantly different from zero indicating the absence of human capital spillovers among regions. Another result of interest is that the impact estimates for the spillover effects arising from changes in population growth are considerably larger than one would infer from the coefficient estimate on the spatial lag variable.

The direct impact for $\ln s_i^K$ is 0.58 and the indirect impact representing spillover effects generated by physical capital accumulation is -0.27. This means that a one percent increase in physical capital investment in all economies will lead to a total impact (= the sum of direct and indirect impacts) of 0.58-0.27=0.31 percent increase in future per worker output on average.

In terms of indirect effects reported in the table, there are two interpretations. One reflects the impact a region has on all other regions, and the other relates the impact of all other regions on a particular region. In terms of the impact a region has on all other regions, a one percent increase in physical capital investment in a region will on average result in all other regions collectively experiencing a 0.27 percent increase in future output. This impact is spread out over multiple regions, and thus individual regions will experience a smaller increase. Since the total impact declines in magnitude with the order of neighbours, the indirect impact will have a greater impact for nearby neighbouring economies.

The other interpretation involves the impact of all other economies on a particular economy. That is, a one percent increase in physical capital investment in all other regional economies will on average lead to a 0.27 percent increase in future output for the regional economy of interest. Although the estimated magnitude of 0.27 is the same in both cases, it matters whether the interpretative focus is on a typical economy's relation to all others (called *impact from an observation*), or all other economies' relative to a typical economy (called *impact on an observation*).

The indirect impact estimates provide clear evidence for the presence of knowledge spillovers generated by physical capital accumulation. The indirect impact estimate of the human capital variable is not significant. This indicates the absence of human capital externalities, and may have different explanations. One is to point to the discrepancy between the theoretical variable representing human capital in the production function and the proxy used for investments in human capital in the empirical model specification. The educational attainment variable is a very partial measure of the rate of investment in human capital, and, more important, does not account for regional differences in the quality of education.

Turning to the implied parameter values in Table 2, we observe larger discrepancies between those based on raw parameter estimates and those based on the impact estimates. The implied value of the parameter θ that reflects spatial connectivity of physical capital per worker within the region becomes larger, while the value of the parameter ρ that reflects the degree of regional interdependence becomes smaller, but still clearly reveals that technological spillovers among regions are far from being negligible.

5 Closing comments

This paper presents and tests a model of economic growth in a world of independent regions where disembodied knowledge diffusion plays an important role in the long-run growth. The stock of knowledge in one regional economy is assumed to produce physical and human capital externalities that may cross regional borders and spill over into other regions with an intensity that decreases as we move to more distant or less connected economies.

When appropriately measuring the direct as well as indirect impacts of changes in MRW variables on output levels we find evidence for the presence of physical capital – but not for the presence of human capital[22] – spatial spillovers. The results, moreover, show that the economies do not evolve independently from each other due to technological interdependence. This finding implies that regional economies cannot be treated as independent observations and growth models should explicitly account for this kind of interdependence.

Our model rests on the existence of a geographic component to the disembodied knowledge spillover mechanism. Conventional wisdom that geographic distance attenuates spillovers supports this assumption. Regardless of geographic proximity, knowledge spillovers are also believed to be higher between regions with similar technological profiles (see, for example, Fischer et al. 2006). According to this view, the ability to make productive use of another region's knowledge depends on the degree of technological similarity between regions. One avenue for future research would be to explore the importance of the technological dimension to the spillover mechanism in regional growth processes.

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[22] This might be due to the fact that the educational attainment variable is only a very partial measure for the theoretical variable representing human capital in the theoretical model.

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Table 1 The spatially augmented MRW model: Constrained and unconstrained estimation results

	The spatially augmented MRW model					
	Constrained estimation			Unconstrained estimation		
	Coeffic.	Standard deviation	*p*-value	Coeffic.	Standard deviation	*p*-value
Constant	2.7900	0.5659	0.0000	5.9666	1.0509	0.0000
$\ln s_i^K$	–	–	–	0.5582	0.0377	0.0000
$\ln s_i^H$	–	–	–	0.1535	0.0319	0.0000
$\ln(n_i + 0.05)$	–	–	–	-0.0873	0.1001	0.3829
$\ln s_i^K - \ln(n_i + 0.05)$	0.5421	0.0411	0.0000	–	–	–
$\ln s_i^H - \ln(n_i + 0.05)$	0.1180	0.0341	0.0005	–	–	–
$W \ln s_j^K$	–	–	–	-0.2768	0.0624	0.0000
$W \ln s_j^H$	–	–	–	-0.1353	0.0409	0.0009
$W \ln(n_j + 0.05)$	–	–	–	0.4387	0.1701	0.0099
$W[\ln s_j^K - \ln(n_j + 0.05)]$	-0.3196	0.0602	0.0000	–	–	–
$W[\ln s_j^H - \ln(n_j + 0.05)]$	-0.1248	0.0431	0.0038	–	–	–
$W \ln y_j$	0.7770	0.0456	0.0000	0.6670	0.0584	0.0000
Implied parameters						
α_K	0.2604	0.0357	0.0000	0.2548	0.0394	0.0000
α_H	0.1018	0.0342	0.0029	0.1257	0.0373	0.0007
θ	0.0659	0.0238	0.0056	0.0714	0.0288	0.0133
ϕ	-0.0310	0.0245	0.2063	-0.0361	0.0276	0.1908
ρ	0.7361	0.0548	0.0000	0.6307	0.0660	0.0000
ξ	0.4203	0.0157	0.0000	0.4404	0.0143	0.0000
Test of restrictions (LR)	–	–		46.3978	(*p*=0.0000)	
Common factor test (LR)	63.8451	(*p*=0.0000)		29.6094	(*p*=0.0000)	
Diagnostics						
Heteroskedasticity	12.6332	(*p*=0.0132)		7.7210	(*p*=0.2593)	
Normality	231.2504	(*p*=0.0010)		23.8075	(*p*=0.0016)	
Performance measures						
Log likelihood/*N*	0.9308			1.0479		
Sigma square	0.0152			0.0128		
R^*	0.9493			0.9660		

Notes: The rates s^K, s^H and *n* are averages over the period 1995-2003; LR denotes likelihood ratio; $\xi = \alpha_K + \alpha_H + (\theta + \phi)(1 - \alpha_K - \alpha_H - \theta - \phi)^{-1}$; standard errors and *p*-values of the implied values of α_K, α_H, θ, ϕ, ρ and ξ are calculated using a simulation method (10,000 random draws); heteroskedasticity is tested using the studentized spatial Breusch-Pagan test, and normality using the Jarque-Bera test; R^* is a measure of the overall fit of the model, defined as the correlation between the fitted and observed values of the dependent variable.

Table 2 The spatially augmented MRW (unrestricted estimation): Direct and indirect impact estimates in comparison with the raw coefficient estimates

	Impact estimates			Raw coefficient estimates [=LS-interpretation]		
	Coeffic.	Standard deviation	*p*-value	Coeffic.	Standard deviation	*p*-value
Direct impacts						
$\ln s_i^K$	0.5801	0.0355	0.0000	0.5582	0.0377	0.0000
$\ln s_i^H$	0.1463	0.0309	0.0000	0.1535	0.0319	0.0000
$\ln (n_i + 0.05)$	-0.0022	0.0988	0.9826	-0.0873	0.1001	0.3829
Indirect impacts [=*spatial spillovers*]						
$W \ln s_j^K$	-0.2653	0.0680	0.0001	-0.2768	0.0624	0.0000
$W \ln s_j^H$	-0.0919	0.0849	0.2791	-0.1353	0.0409	0.0009
$W \ln (n_j + 0.05)$	1.0717	0.4268	0.0120	0.4387	0.1701	0.0099
Implied parameters						
θ	0.9044	0.2024	0.0000	0.0714	0.0288	0.0133
ϕ	-0.0893	0.1640	0.5864	-0.0361	0.0276	0.1308
ρ	0.2854	0.0549	0.0000	0.6307	0.0660	0.0000

Notes: To obtain the impact estimates we simulated 10,000 instances of $\boldsymbol{y}$, and estimated the parameters for the spatial Durbin model specification via maximum likelihood. Using the set of 10,000 estimates, we used LeSage and Pace's (2009) efficient formulas to compute the average direct and indirect impacts along with the standard deviation of the 10,000 outcomes. The table shows the average over the 10,000 impact estimates along with the associated standard deviation and *p*-values.