Spatial structures in spatial autoregressive models and one-directional adjacency matrices

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In the context of spatial econometrics, we propose a new idea, the specification of one directional effects, not mutual dependencies, and try to utilize this concept in a land price model. Spatial (or network) interdependency should exist almost everywhere in the real world. We rarely use econometric models with spatial autoregressive structures, however. Using an empirical study (maximum likelihood estimations of a spatial autoregressive model of land price data in Miyagi Prefecture, Japan), we show that the spatial dependencies may not be recognized if we assume that such dependencies are reciprocal.

Keywords: *Land price, Spatial autocorrelation, Spatial adjacency matrix*

JEL Classifications: *R14, C21*

1 Introduction

Spatial autoregressive models, which were applications of time-series models at first, are with some modeling of spatial (network) covariance structure, which is often neglected. We generally assume mutual interdependencies in a spatial autoregressive model. In some cases, it is rather natural to think there are only one-directional dependencies, not mutual dependencies. Some researches, land price spillover model Ando and Uchida (2004) for an example, adopted concepts similar to one-directional dependencies. In this paper, we first categorize adjacency matrices which were used in existing researches. We then propose a new idea, specifications of one directional effect, not mutual dependencies and try to utilize the concept to a land price model. Using an empirical study (maximum likelihood estimations of a spatial autoregressive model of land price data in Miyagi Prefecture, Japan), we show that the spatial dependencies may not be recognized if we assume that such dependencies are reciprocal.

2 Spatial autoregressive models and spatial adjacency matrices

2.1 One-directional adjacency matrices

There is an implicit assumption that the interdependency is mutual. Anselin and Bera (1998, p.245) pointed out that,

> While no inherently invalidating estimation or testing procedures, the unconnected observations imply a loss of degrees of freedom, since, for all practical purposes, they are eliminated from consideration in any "spatial" model.

The model becomes just an ordinal regression model, because the isolated sample have no effects on others.

We think it is natural to assume that some of sample affect others and are not be affected. For example, the land price in the CBD of a large metropolitan area is determined independently and these of other area of the metropolitan area depend on that of CBD. Rincke (2006) pointed out possibilities of such relationships.

We apply this idea of one-directional influence on a land price model. Land prices are determined mainly by the location attributes. Other important factor is the prices in surrounding areas. Speculations determine some part of land prices. Forecasts of the price of land will receive big influences from prices of land in neighborhood. However, if a price of land is relatively low, it may be disregarded as a judging material of the forecast.

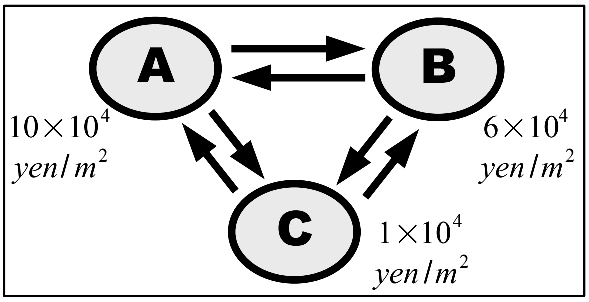

Fig.1-1: $h = 0$

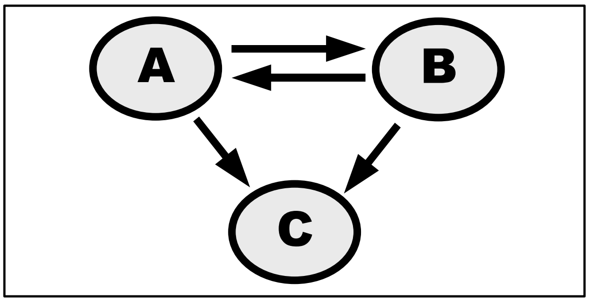

Fig.1-2: $h = 1/2$

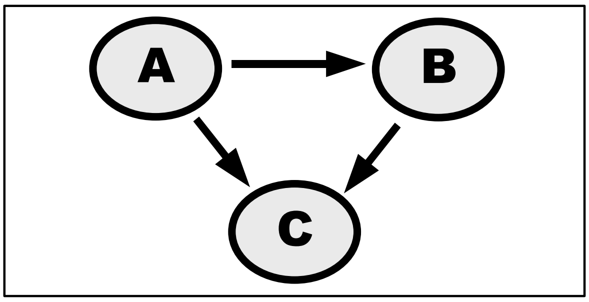

Fig.1-3: $h = 1$

Figure 1: Land price interaction and it's threshold h

We introduce a simple economic model of land prices. There are n locations. We assume interdependency in land prices of the locations $\bar{y}_i$. We also assume there is a threshold h in the interdependency. The influence on location i from location j exists if the relative price of j ($\bar{y}_j/\bar{y}_i$) is higher than h.

3 An empirical application

In this section, we utilize the idea of one-directional adjacency matrices to an example of empirical analysis, a land price model in a Japanese region (Miyagi prefecture in north-east part of Japan). We assume land prices are determined by two aspects. The first aspect represents real economy attributes, floor-area-ratio for example. The second represents psychological or speculative one, land prices in neighboring areas. It is more natural to assume that the land price in a location is affected not by every price in surrounding area, but only by higher prices. We assume that the relative price in the previous year determines whether there is influence from one location to another. Land prices in the previous year are exogenous variables.

3.1 Data and model

We use land price notification statistics provided by Japanese national government (`http://nlftp.mlit.go.jp/ksj/`). Explained variables are notified housing land price in Miyagi prefecture at 2005. 452 locations, whose data for both 2004 and 2005 are available, are chosen as our sample. We employ following variables for each location i.

y_i: Notified housing land prices at 2005 (Yen/m^2)

$\bar{y}_i$: Notified housing land prices at 2004 (Yen/m^2)

$x_{i1} = 1$: Constant term

x_{i2}: Distance from the prefectural center (m)

x_{i3}: Distance from the nearest railroad station (m)

x_{i4}: Upper limit of floor-area-ratio (%)

x_{i5}: Dummy variable of gas supply

Descriptive statistics for each variable are shown in Table 1.

Our spatial autoregressive model is represented in matrix form as,

$$\boldsymbol{y} = \rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},\ \ \boldsymbol{\epsilon} \sim \mathrm{IIN}(0, \sigma^2). \tag{1}$$

Table 1: Descriptive statistics

Variable		Mean	S.D.	Min	Max	Unit
y_i	Land price at 2005	56760	28390	4000	183000	Yen/m^2
$\bar{y}_i$	Land price at 2004	60482	29576	4100	188000	Yen/m^2
x_{i2}	Distance from the prefectural center	18120	19800	0	101300	m
x_{i3}	Distance from the nearest station	2982	4115	90	36600	m
x_{i4}	Floor-area-ratio	162.8	58.86	60	400	%
x_{i5}	Gas supply	0.677	0.4681	0	1	

The Log likelihood function is derived as

$$\boldsymbol{A}\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},\ \ \boldsymbol{A} = \boldsymbol{I} - \rho\boldsymbol{W}, \tag{2}$$

$$\mathcal{L} = -\frac{n}{2}\log(2\pi) + \log|\det\boldsymbol{A}| - \frac{n}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum(\boldsymbol{A}\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})'(\boldsymbol{A}\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}). \tag{3}$$

3.2 Specification of $\boldsymbol{W}$

We adopt an inverse-distance matrix as the spatial adjacency matrix $\boldsymbol{W}$ and its elements w_{ij} are calculated by following equations ($\alpha > 0, i, j = 1, \cdots, N$).

$$w_{1ij} = \begin{cases} 1\big/d_{ij}^{\alpha} & \text{if } i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$w_{0ij} = w_{1ij}\Bigg/\sum_{k=1}^{N} w_{1ik} \tag{5}$$

$$w_{ij} = \begin{cases} w_{0ij} & \text{if } i \neq j, \bar{y}_j/\bar{y}_i \geq h \text{ and } d_{ij} < 40\text{km} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

d_{ij} is the distance between location i and j. α is a parameter for the distance decay. h is a threshold parameter of land price influences. Equation(4) represents the assumption that magnitudes of influence between two areas depend on their distances. Equation(5) represents the assumption that the total amount of influence one area received is fixed.

Table 2: Estimation results($\alpha = 0.5$)

α	h	$\mathcal{L}$	ρ	σ^2	β_1	β_2	β_3	β_4	β_5
0.5	0.	-4931.79	0.00127517	3.51366×10^8	38023.1	-0.66706	-1.25435	127.512	20292.5
			(0.00643)	(15.03)	(2.495)	(-4.569)	(-5.624)	(7.891)	(7.8)
0.5	0.25	-4918.04	0.696549	3.2917×10^8	-10766.5	-0.230505	-0.949157	130.858	18738.5
			(5.736)	(15.03)	(-1.123)	(-2.302)	(-4.322)	(8.366)	(7.805)
0.5	0.5	-4840.52	0.996932	2.32227×10^8	-11930.4	-0.0892403	-0.0112522	99.6394	7907.73
			(22.06)	(15.03)	(-2.585)	(-1.475)	(-0.05772)	(7.571)	(3.764)
0.5	0.75	-4704.13	1.1225	1.27708×10^8	15995.2	-0.177404	0.0110566	29.9112	-422.042
			(33.08)	(15.03)	(6.337)	(-4.563)	(0.07836)	(2.899)	(-0.2548)
0.5	1.	-4542.29	1.28818	6.27012×10^7	37126.	-0.321311	-0.278352	-3.18277	-1008.07
			(46.14)	(15.03)	(21.98)	(-12.71)	(-2.887)	(-0.4297)	(-0.886)
0.5	1.25	-4444.71	1.58306	4.07156×10^7	45445.7	-0.410882	-0.539491	-3.67978	2927.76
			(58.47)	(15.03)	(33.3)	(-20.52)	(-7.011)	(-0.6201)	(3.289)
0.5	1.5	-4496.6	1.93625	5.12255×10^7	46977.5	-0.481577	-0.715901	14.8825	6919.23
			(52.22)	(15.03)	(30.64)	(-21.65)	(-8.335)	(2.27)	(7.05)
0.5	1.75	-4591.74	2.38825	7.8039×10^7	46254.5	-0.523631	-0.840652	33.3742	10449.9
			(41.56)	(15.03)	(24.43)	(-19.15)	(-7.953)	(4.164)	(8.731)
0.5	2.	-4661.02	2.88121	1.0603×10^8	45012.5	-0.563357	-0.913729	51.4696	12736.7
			(34.77)	(15.03)	(20.42)	(-17.73)	(-7.428)	(5.578)	(9.209)

Note：Numbers in parentheses are t values.

In Equation(6), the threshold of relative price determines there is influence or not in each pair. Please notice that row totals of $\boldsymbol{W}$ may be 0 in some cases. But row totals of the matrix $\boldsymbol{A} = \boldsymbol{I} - \rho\boldsymbol{W}$ are always non-zero.

3.3 Estimation results

We estimate our empirical model with the maximum likelihood method. [1] Although α is fixed at 0.5, we examine several values of threshold h, 0 through 2. In other words, we estimate several empirical models for each $\boldsymbol{W}$. Table 2 shows the log-likelihood and estimators for each $\boldsymbol{W}(h)$. Each row corresponds to a different value of h. Please pay your attention to two cases, every pairs have a interdependent relationship ($h = 0$) and locations are affected only by the location where land prices are at least same level ($h = 1$). The former, $h = 0$, is widely assumed implicitly in the literature. In our result, the latter,

[1] See Anselin (1988) for the derivation of the estimated covariance matrix here.

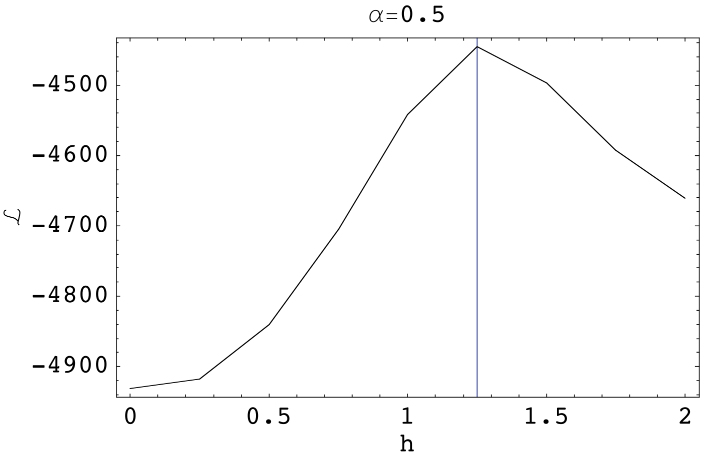

Figure 2: h and log-likelihood

$h = 1$, is with much higher log-likelihood. Figure 2 depicts the relationship between h and likelihoods. The log likelihood becomes larger with h until $h = 1.25$.

There are remarkable differences in estimators of β's. In the case of a ordinal spatial weight matrix, $h = 0$, each coefficients of parameters are significant and have theoretically correct signs. β_2: the coefficient of "distance from the prefectural center" is significantly negative. β_3: the coefficient of "distance from the nearest railroad station" is also significantly negative. The coefficients of β_4: "upper limit of floor-area-ratio" and β_5: "dummy variable of gas supply" are significantly positive. In the case of a one-directional spatial weight matrix, $h > 0$, some of β's are not significant.

It is clear that the significance of ρ, the coeficient of spatial term, depends on h. It is not significantly non-zero in the case of a ordinal spatial weight matrix, $h = 0$. But it is significantly positive for a larger value of h.

This empirical example shows that the implicit assumption of mutual dependency may be too strong. The explanatory powers of models are larger and results about significances of parameter become different in cases of a one-directional spatial weight matrix, $h > 0$.

4 Conclusion

In the context of spatial econometrics, we propose a new idea, the specification of one directional effects, not mutual dependencies, and try to utilize this concept in a land price model. Using an empirical study (maximum likelihood estimations of a spatial autoregressive model of land price data in Miyagi Prefecture, Japan), we show that the spatial dependencies may not be recognized if we assume that such dependencies are reciprocal.

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