Federal Reserve Policy and US State Income Dynamics[1]

Hua Kiefer
Comptroller of the Currency
250 E Street, SW
Washington DC 20219
email: hua.kiefer@occ.treas.gov

and

Leonard Kiefer

Version: May, 2009

This study examines the impact of changes in the Federal Reserve target interest rate on state income dynamics. We show that there is considerable contemporaneous correlation between state income growth rates, and that this correlation exhibits a spatial pattern. Without considering contemporaneous income correlations, estimates of impulse response functions at the state level are likely to be biased. Impulse response functions and variance decompositions show considerable variation between states, but this variation decreases markedly when examining the states by region. This shows that there is substantial heterogeneity across states in the channels through which monetary policy propagates. The existence of spillover effects point to an additional channel not clearly identified heretofore in the literature.

Key Words: Monetary policy, Spatial effects, Panel data
Subject Classification: [JEL Classification] C31; E58; E52; R15

1. INTRODUCTION

A substantial literature in macroeconomics devotes itself to asking whether or not, and to what extent monetary policy matters. A broad consensus has emerged that in the short run monetary policy can have an impact on real income. Numerous studies have devoted themselves to examining the relationship between monetary policy actions and real income. The emerging consensus is that monetary policy actions have a delayed and persistent effect on real income dynamics.

A related, but much smaller literature has looked at the disaggregated impact of monetary policy. This literature asks how national level policies affect regional economies. These studies have found considerable heterogeneity in regional and state level income dynamics in response to monetary policy. It is important to recognize that though monetary policy is set at the national level, its impact is felt locally. Tightening or expanding credit can have various effects on regional and state level economic conditions. Furthermore, Fed policy is set at the national level and must respond to aggregate conditions. This does not mean that regional conditions do not matter (or necessarily that they are ignored by policy makers). Indeed, the Federal Reserve system publishes Beige books which summarize regional economic conditions. These regional conditions

[1] The views expressed in this paper are those of the authors alone and do not reflect those of the Office of the Comptroller of the Currency.

inform the decisions of policymakers, but only insofar as they give insight into the aggregate economic picture. Since policymakers can only set a single instrument, such as the Federal Funds rate which applies to all regions at the same time, policymakers are constrained to effectively ignore regional heterogeneity when setting policy. Although the Fed can not target at the economy of a specific state, knowing the dynamic pattern of how monetary policy propagate through states still fortifies the efficacy of its policy action.

What is unclear is to what extent do state and regional economic conditions matter for the impact of policy on aggregate income. Studies that focus on the national level necessarily assume that heterogeneity at the regional level do not matter much for the aggregate response. Many dynamic general equilibrium models only consider a single sector's response to monetary policy. And statistical models, typically vector autoregressions (VARs), use only national level income statistics. So though the literature in structural models of monetary policy has come to something of a consensus on the impact of monetary policy on the aggregate economy, it is unclear to what degree regional heterogeneity may impact the aggregate response.

Although there are researches in the literature of regional heterogeneity studying the asymmetric effects of monetary policy on regional economies (Carlino and DeFina, 1998 and 1999, Owyang and Wall, 2003, Sill, 1997), the scope of these researches was restricted to understanding the difference in regional responses and their possible causes.[2] There are few studies that link the disaggregated heterogeneity to the aggregate economic dynamics in response to a Federal policy (Fratantoni and Schuh, 2003).[3] However, in studying the dynamics of how monetary policy propagates through states, the existing literature assumes away the contemporaneous interaction of regional feedbacks in local economic activities. In other words, previous studies suggest that monetary policy affects local economies only directly though either an interest rate channel or a credit channel. Thus, if a state has weak interest rate and credit channels in connection with monetary policy, the influence of a policy action is expected to be small. Suppose there exists contemporaneous spillover effects between states' economic activities, even state i is not sensitive to monetary policy, as long as it is near another state j that is economically sensitive to the Fed's policy, the influence of a policy action on state i is thus expected to be large. Obviously, the existence of spillover effects alter the dynamic pattern of local economies in response to policy shock.

In this paper, we argue that economic shocks originated in one state may directly affects the economy in the nearby states within the same time period. For example, a shock to a production, such as a hurricane or a flood, that hits Louisiana will also impact Texas as well because of the proximity of these two states. Thus, the agricultural production of both states become would likely decline. When homeowners in California started to lose money because of the declining housing market there, house prices in Nevada and Arizona also plummeted and construction incomes in these states fell quickly.[4] The decrease of homeowners' wealth is translated into a negative consump-

[2] There are two conventional ways to explain the various economic sensitivities of states in response to monetary policy, including an interest rate channel: due to differing interest rate elasticities of industries coupled with differing geographical concentrations of industries; and a credit channel: due to the degree to which firms are dependent on banks for credit (Bernanke and Blinder, 1988; and Kashyap et al., 1993) and the ease with which banks can adjust their balance sheets (Kashyap and Stein, 1995).

[3] Fratantoni and Schuh estimate the impact of regional heterogeneity in housing markets at the MSA-level for on the efficacy of monetary policy. They find that the aggregate impulse response to a contractionary policy shock exhibits nonlinear properties, which result from the heterogeneity of the initial regional economies over MSAs.

[4] Home Price Index (HPI) released by First American LoanPerformance indicates that for a 12 month time period between 09/2006 and 09/2007, California declines by 12.65%, Nevada declines by 9.34%, and Arizona decreases by 7.3%.

tion demand shock. Not only consumers in California, but also those in nearby states, tend to modify their consumption behavior due to the pessimistic expectations of the future, which induces correlated economic outcomes in these states. Another example demonstrating contemporaneous neighborhood effects across states is local government spending behavior. Yard stick competition, local governments tend to compete with their neighbors on public expenditure, is well recognized in public finance literature. A spending increase of intrastate highway projects in Maryland may provoke Virginia to increase its spending on infrastructure. In general, spillovers of states' economic activities, which might be initiated by diverse causes, can be widely observed throughout the time.

This study examines the impact of Federal Reserve Board monetary policy on state level income dynamics. We find evidence for the existence of contemporaneous correlation between states' economic activities, and this correlation has a spatial component. This implies that when studying income dynamics at the state level, a restriction suppressing contemporaneous correlations between state incomes may lead to misestimated dynamics at the national level that provide inaccurate information about the aggregate economic picture to policymakers. If state income dynamics are significantly shaped by contemporaneous neighborhood effect, policymakers might want to identify this influence in formulating policies and assessing their consequences. Furthermore, like previous studies we find a large degree of regional heterogeneity in responses to monetary policy.

In particular, we initiate our investigation of the income dynamics in the 48 contiguous U.S. states with a structural vector autoregression (SVAR) which contains both national and state-specific variables. The reduced econometric form of the SVAR breaks down into a state-level spatial autoregressive (SAR) model and a national-level aggregate model. Our empirical analysis of the reduced model exploits quarterly measures of real income growth between the period of 1975 Q1 and 2001 Q4. In the state-level SAR model, we include a component of contemporaneous neighborhood effects allowing interactions between states' economic activities, which raises an endogeneity issue. To overcome the endogeneity problem, a maximum likelihood (ML) estimator is employed in estimating the state-level spatial model.

This paper is organized as follows. Section 2 outlines the basic economic model specifications. Section 3 describes the data. Section 4 provides the technical details of the reduced econometric models. The empirical results and the interpretations of the results are presented in Section 5. The last section offers conclusions and extensions.

2. ECONOMIC MODEL

A structural vector autoregression (SVAR), which accounts for feedbacks between all system variables in identifying the effects of policy shocks, has the following form

$$A_0 X_t = M + A_1 X_{t-1} + A_2 X_{t-2} + \cdots + A_q X_{t-q} + U_t, \tag{1}$$

where M is a vector of K constant parameters, X_t is a covariance-stationary vector with K system variables in period t, A_0 represents the $(K \times K)$ coefficient parameter matrix describing the contemporaneous correlations among the variables, $A_i s$ denote the $(K \times K)$ coefficient parameter matrices describing the lagged correlations among the variables, q is the number of time lags, and U_t is the vector of structural disturbances with a dimension of K. Equation (1) can also be written in a form of lag operator as

$$A(L) X_t = M + U_t,$$

where $A(L) = A_0 - A_1L - A_2L^2 - \cdots - A_qL^q$ is a matrix polynomial in the lag operator of order q. If A_0 is nonsingular, the reduced form of the model - upon which the estimation is based - is:

$$\begin{aligned} X_t &= A_0^{-1}M + A_0^{-1}(A_1X_{t-1} + A_2X_{t-2} + \cdots + A_qX_{t-q}) + A_0^{-1}U_t \\ &= b + B_1X_{t-1} + B_2X_{t-2} + \cdots + B_qX_{t-q} + V_t \\ &= b + B(L)X_{t-1} + V_t, \end{aligned} \tag{2}$$

where $b = A_0^{-1}M$, $B_1 = A_0^{-1}A_1, B_2 = A_0^{-1}A_2, \cdots B_q = A_0^{-1}A_q$, $V_t = A_0^{-1}U_t$, and $B(L) = B_1L + B_2L^2 + \cdots + B_qL^q$. Although we can use the estimates of the reduced form parameters to carry out forecasting exercises, we can not use them to identify the impact of policy shocks. Because disturbances in the reduced form are linear combinations of the structural shocks, the reduced model is not a suitable framework for policy analysis. It is easier to see the impact of structural shocks in a moving average (MA) representation as

$$X_t = A(L)^{-1}(M + U_t) = c + C(L)U_t,$$

where $C(L) = A(L)^{-1}$is an infinite-order lag matrix polynomial,[5] describing the relationship between the system variables and the structural innovations. The terms in matrix polynomial $C(L)$ can be directly interpreted as the impulse responses of system variables to the structural shocks. For example, if the jth element of the structural disturbance vector U is subject to a shock equal to one standard deviation at time t, the effect felt by the ith system variable in X at time $t+s$ is measured by the coefficient of L^s in the ijth element of the $C(L)$. Suppose the jth element of U represents a monetary policy shock, in order to generate the impulse response function of the structural monetary policy shock, we will need to identify the coefficients in the polynomial of the ijth element of the $C(L)$, which is a function of the parameter matrices As.

Let $n = 48$ indicating the total number of contiguous states,[6] we apply this SVAR framework to our analysis of economic activity in these 48 states and define X as

$$X_t = [y_{1t}, y_{2t}, \cdots, y_{48t}, p_t, r_t]' = [y'_{nt}, p_t, r_t]',$$

in which $y_{nt} = [y_{1t}, y_{2t}, \cdots, y_{48t}]'$, and vector X_t includes 50 system variables. The variable $y_{it} (i \in [1, 48])$ is the growth rate of real incomes in state i at time t, r_t is the monetary policy instrument measured by the effective federal funds rate at time t, and p_t is the inflation rate of at time t. Accordingly, the structural disturbance vector, U_t, has the form

$$U_t = [u_{1t}, u_{2t}, \cdots, u_{48t}, u_{pt}, u_{rt}]' = [u'_{nt}, u_{pt}, u_{rt}]',$$

where $u_{1t} = [u_{1t}, u_{2t}, \cdots, u_{48t}]'$, u_{it} $(i \in [1, 48])$ denotes the state-specific innovation, u_{pt} denotes the structural disturbance in inflation rate, and u_{rt} denotes the structural monetary policy shock. The corresponding disturbance term in the reduced form of equation (2) is

$$V_t = [v_{1t}, v_{2t}, \cdots, v_{48t}, v_{pt}, v_{rt}]' = [v'_{nt}, v_{pt}, v_{rt}]'.$$

Clearly, each of the system variables in X_t including state-level income growth, federal funds rate, and inflation rate can be influenced by not only its own idiosyncratic

[5] Matrix $C(L)$ has a dimension of $K \times K$, in which each entry is an infinite order of lag operators. For example the ijth element of $C(L)$ has the form of $C_{ij}(L) = \alpha + \beta L + \cdots + \gamma L^\infty$.

[6] Alaska and Hawaii are not contiguous to any of the other state, and therefore excluded from this analysis.

shocks, but also the shocks to other variables. For example, the real income growth in the state of Virginia can be influenced by the income shock to Virginia, the income shock to any other state, the national monetary policy shock and the shock to the national inflation rate. The matrix A_0 determines how shocks are transmitted into the contemporaneous variables, and the matrices $A_i s$ determine how shocks are dispersed into the system through the lagged subsequent periods.

3. DATA

This study employs quarterly data on real income by state for the period of 1975 Q1 to 2001 Q4. To compute real income growth,[7] we use the data from Bureau of Economic Analysis (BEA), including personal nominal income data and GDP deflator, which is the ratio of real GDP[8] and nominal GDP. **Table 1** presents the summary statistics of real income growth rate for the 48 US states, which are grouped into 8 geographic regions defined by BEA.[9] The distribution of real income growth indeed displays variations across states, which gives rise to the heterogeneous responses of subnational economies to a policy shock. The quarterly effective federal funds rate data is acquired from Board of Governors of the Federal Reserve System.[10] The quarterly inflation rate at the national level is computed using consumer price index (CPI) collected from the website of Bureau of Labor Statistics (BLS).

[7] The real income is valued in thousands of 2000 dollars.

[8] The real GDP is valued in billions of 2000 dollars.

[9] The regional classifications, which were developed in the mid–1950's, are based on the homogeneity of the states in terms of economic characteristics, such as the industrial composition of the labor force, and in terms of demographic, social, and cultural characteristics. For a brief description of the regional classification of states used by BEA, see U.S. Department of Commerce, Bureau of the Census, Geographic Areas Reference Manual, Washington, DC, U.S. Government Printing Office, November 1994, pp. 6–18–6–19.

[10] We have aggregated the monthly data on federal funds rate to a quarterly one.

Table 1: US State Real Income Growth Rate

(48 contiguous states in the U.S. over the period 1975 Q1 – 2001 Q4)

New England Region	Pct*	*Southeast Region*	Pct	*Southwest Region*	Pct
Connecticut	0.88	Alabama	0.79	Arizona	0.79
Maine	0.48	Arkansas	0.81	New Mexico	0.71
Massachusetts	0.82	Florida	0.83	Oklahoma	0.97
New Hampshire	1.11	Georgia	0.78	Texas	0.87
Rhode Island	0.98	Kentucky	0.53	*Rocky Mountain Region*	
Vermont	1.11	Louisiana	0.68	Colorado	0.70
Mideast Region		Mississippi	0.43	Idaho	0.61
Delaware	0.77	North Carolina	0.99	Montana	0.79
District of Columbia	**	South Carolina	0.74	Utah	0.60
Maryland	0.66	Tennessee	1.13	Wyoming	1.03
New Jersey	0.71	Virginia	0.76	*Plains Region*	
New York	0.75	West Virginia	1.58	Iowa	0.61
Pennsylvania	1.26	*Far West Region*		Kansas	0.56
Great Lakes Region		Alaska	**	Minnesota	0.64
Illinois	0.57	California	1.11	Missouri	0.65
Indiana	0.58	Hawaii	**	Nebraska	0.76
Michigan	0.69	Nevada	1.14	North Dakota	0.91
Ohio	0.89	Oregon	0.52	South Dakota	4.09
Wisconsin	0.86	Washington	0.92		

*The percentage of growth rate is proximated by the log difference of real incomes times 100.

**The growth rates of District of Columbia, Alaska, and Hawaii are not used in the estimation, and not reported here.

The measure of state income growth, y_{it}, used in the estimation must be stationary so that standard statistical theory applies. We apply an augmented Dickey-Fuller (ADF) test to check for a unit root in our data sample. The stationary test is carried for each state in both the level and the growth rate of income. We state our unrestricted models as

$$y_t = \alpha_1 + \beta_1 t + \gamma_1 Y_{t-1} + \delta_1 y_{t-1} + \theta_1 y_{t-2} + \varepsilon_t, \tag{3}$$

where y_t refers to the **growth rate** (log difference) of state income,[11] t refers to a time trend, and Y_{t-1} refers to the **level** of state income; and,

$$\Delta y_t = \alpha_2 + \beta_2 t + \gamma_2 y_{t-1} + \delta_2 \Delta y_{t-1} + \theta_2 \Delta y_{t-2} + \varepsilon_t, \tag{4}$$

where Δy_t refers to the change of growth rate of state income. Since Y is the level, y becomes the lagged first difference of Y (log difference), and Δy is thus the lagged second difference. The restricted models have γs restrict to be zeros in equation (3) and (4).

The test results show that we can not reject for null for equation (3) at even the 10% level of significance for any state. In contrast, the null of equation (4) is rejected for every state at the 1% level.[12] These results suggest that the growth rate of state income is stationary, but not the level of income. Note that the specification for equation (3) and (4) includes two time lags of each variable, our choice on the number of lags is based on the Bayesian Information Criterion (BIC), which indicates that two lags are enough to eliminate serial correlation from the residuals.

[11] As the test is carried for each state separately, the state subscript i is suppressed.

[12] The ADF test statistics follow a τ distribution. The critical value(CV) of rejecting the null at the 1% level of significance is -4.15, the one at the 5% level is -3.5, and the one at the 10% level is -3.15.

4. ECONOMETRIC MODEL

4.1. Identification and Simplifying Assumptions

Comparing equation (1) and equation (2), the full knowledge of A_0 would allow us to recover estimates of the other As and the structural innovations from the estimated parameters of Bs. We could then proceed to evaluate the role played by the monetary policy shock as the driving force behind movements in state income growth. With only 25 years of quarterly data it is impossible to estimate an unrestricted version of this model. Indeed, even with ample data, we would be unable to fully identify all of the parameters. In order to ensure that the reduced form economic system can be mapped onto the structural disturbances we need to make several assumptions. These assumptions follow the tradition of recursiveness assumption (see e.g. Christiano, Eichenbaum, and Evans 1998) and assumes that certain economic variables respond to innovations to other variables only with a lag. These assumptions show up as restriction on the matrix A_0. These assumptions are as follows:

1. Regional income growth responds to the price level and interest rate only with a lag.
2. The price level responds contemporaneously to state income growth, but only responds to the interest rate with a lag.
3. The state income growth rates are interdependent based on a predetermined weight matrix.
4. The price level and interest rate only respond to the weighted average level of state income growth.

We present the detailed structure of matrix A_0 to reflect the first two imposed assumptions as,

$$A_0 = \begin{array}{c} \\ y_{1t} \\ \vdots \\ y_{48t} \\ p_t \\ r_t \end{array} \begin{array}{c} \begin{matrix} y_{1t} & \cdots & y_{48t} & p_t & r_t \end{matrix} \\ \begin{bmatrix} 1 & \cdots & ? & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & 0 \\ ? & \cdots & 1 & 0 & 0 \\ ? & \cdots & ? & 1 & 0 \\ ? & \cdots & ? & ? & 1 \end{bmatrix} \end{array}, \tag{5}$$

where 0s represent the implication of those assumptions, and ?s denote all possible values without any restriction. Assumption 1 and 2 help to reduce the number of parameters for estimation in matrix A_0 by 97 from 2500 (50^2) to 2403. Assumption 3 further imposes restrictions on the nonzero entries of the first 48 rows in matrix A_0, and the last assumption puts additional restrictions on the values of elements represented by ?s in the last two rows.

All four assumptions imply that the matrix A_0 can have the following block recursive form:

$$A_0 = \begin{bmatrix} A_0^{11} & 0_{48,1} & 0_{48,1} \\ -\lambda^p \widehat{W} & 1 & 0 \\ -\lambda^r \widehat{W} & -\delta & 1 \end{bmatrix}, \tag{6}$$

where $0_{48,1}$ indicates a 48×1 column vector of zero entries, and the submatrix A_0^{11} is a 48×48 matrix denoting the contemporaneous responses across states. Assumption 3 imposes restrictions on A_0^{11} through the state income equations, which will be carefully explained in the following sub-section. The 1×48 row vector $\widehat{W}$ is simply a weight vector and works to average the state regional income growth according to the size of regional economy. Finally the parameters λ^p, λ^r, and δ are scalar unknowns that are to be estimated. This imposes an additional 96 restrictions, including the unit restrictions on the last two diagonal elements, and reduced the number of parameters to be 2307 ($A_0^{11}, \lambda^p, \lambda^r$, and δ).

Different from the assumption made by Christiano & etc. (1999) and Carlino & DeFina (1998) regarding regional economic shocks, we allow a state-specific shock to have contemporaneous spillover effects on other states. It is natural and reasonable to take these ripple effects into account when the dynamic response to the monetary policy actions is measured. The existing literature fails to account for these simultaneous movements of subnational economies initiated by a policy shock, and model the regional economic activities in a limited and over-simplified way. In this study, we remove the restriction on the contemporaneous neighborhood interactions in regional economic activities placed by existing studies, and explicitly estimate how the ripple effects influence the aggregate economic response to a contractionary monetary policy in terms of duration and magnitude.

As in the left corner of parameter matrix (6), A_0^{11}, indicates how state economies are correlated. For identification purpose, we apply Assumption 3 to restrict A_0^{11} through the state income equations. The interaction pattern of state incomes are determined in a prior fashion in accordance with BEA's regional definition. Usually, states in a geographic region have a similar composition of industries and homogenous social economic characteristics. Therefore, states are more highly interdependent within a region than over the borders. In addition, commonly shared regional geographic properties fasten the economic relationship among regional states even tighter. A flood in Mississippi tends to influent Louisiana more than Montana. A tightening in the highway budget of New York government is more relevant to the highway budget of New Jersey local government than that of New Mexico government. An intervention in car industry is more likely to boost the economy in Ohio and Michigan than that of Utah and Wyoming. To formulize the extent of states' interaction, we construct a binary square weight matrix, W_n of dimension $n \times n$, where $n = 48$ representing the total number of contiguous states, to define the importance of neighborhood effects over state borders. First, let $k \in \{1, 2, \cdots, 8\}$ denote the specific region where state $i \in \{1, 2, \cdots, 48\}$ is located. The number of states in region k is defined as m_k, and $\sum_{k=1}^{8} m_k = n = 48$. Second, we establish the general rule of weight assignment: if state i and state j are within a same region k, the weight of j's influence on i is set to be equal to the weight of i's influence on j at the value of one; if state i and state j are located in 2 different regions, the weights of the influence between i and j are set to be zeros in either direction.[13] For simplicity, we define state i's *neighboring states* as all the states except for state i in the same region as where state i is. Mathematically, let the element in the i^{th} row and j^{th} column represents the influence of state j on state i. As long as state j is located in the same region as where state i is, the ij^{th} element of W_n receives a constant value of one, suggesting any neighboring state of state i influences state i equally; but if state i and state j are located in different regions, the ij^{th} element of W_n receives a value

[13] This binary matrix is one of the simplest weight matrices, which consists of zeros and ones. The neighborhood influence is simplified to Yes (1) or No (0) without gradual decay. We use it here simply to demonstrate the possible existence of neighborhood effects. Other types of weight matrices, such as contiguity matrices and matrix based on input-output relationship between states are the research interests in the future studies but the in the scope of this paper.

of zero, which implies that neighborhood effects do not extend over regional borders. Each element of W_n's ith row denotes the individual influence of each of the 48 states on state i. Thus, after row normalization, the aggregated neighborhood effect on state i is simply an arithmetic mean of its neighboring states.

As the states have been sorted regarding to the region which they belong to, say, the first 6 states are in New England Region, the next 6 states are in Mideast Region, and etc., W_n is therefore defined as a block-diagonal weight matrix with off-diagonal blocks to be zero sub-matrices. W_n's diagonal blocks denote neighborhood effects within each region and its off-diagonal blocks reflect neighborhood effects across the regional borders. In each diagonal block, the off-diagonal elements have an equal value of ones (neighbors receive same weight); and the diagonal elements are all zeros (state i is not its own neighbor: $W_{ii} = 0$).

The weight matrix W_n is used to aggregate neighborhood effects in a prior fashion. The magnitude of the aggregated neighborhood effects on a given state still relies on the parameter in front of the neighborhood effects. As of the heterogeneity of states over regions, we allow the neighborhood effects to vary across regions. The parameter matrix describing the contemporaneous relationship between states can thus be written as,

$$A_0^{11} = (I_n - \Lambda W_n), \tag{7}$$

where W_n is the predetermined weight matrix, I_n is an identity matrix, and Λ is the spatial auto-regressive coefficient matrix measuring the magnitude of the contemporaneous neighborhood effects. The number of unknown parameters in A_0^{11} depends on the specification of Λ, which will be discussed in the following section.

For simplicity, we assume the number of lags, q, is equal to 2, which implies that two lags of variables are used in the estimation of the SVAR system. After applying the above specifications to the general system of equation (1), our structural economic model has a form of

$$\begin{aligned} \begin{bmatrix} I_n - \Lambda W_n & 0_{48,1} & 0_{48,1} \\ -\lambda^p \widehat{W} & 1 & 0 \\ -\lambda^r \widehat{W} & -\delta & 1 \end{bmatrix} \begin{bmatrix} y_{nt} \\ p_t \\ r_t \end{bmatrix} &= \begin{bmatrix} \Phi_1 + \Pi_1 W_n & \Gamma_1 & \Psi_1 \\ \pi_1^p \widehat{W} & \gamma_1^p & \psi_1^p \\ \pi_1^r \widehat{W} & \gamma_1^r & \psi_1^r \end{bmatrix} \begin{bmatrix} y_{nt-1} \\ p_{t-1} \\ r_{t-1} \end{bmatrix} \\ &+ \begin{bmatrix} \Phi_2 + \Pi_2 W_n & \Gamma_2 & \Psi_2 \\ \pi_2^p \widehat{W} & \gamma_2^p & \psi_2^p \\ \pi_2^r \widehat{W} & \gamma_2^r & \psi_2^r \end{bmatrix} \begin{bmatrix} y_{nt-2} \\ p_{t-2} \\ r_{t-2} \end{bmatrix} \\ &+ \begin{bmatrix} \mu_n \\ \mu^p \\ \mu^r \end{bmatrix} I_{50} + \begin{bmatrix} u_{nt} \\ u_{pt} \\ u_{rt} \end{bmatrix}, \end{aligned} \tag{8}$$

where the coefficient parameter matrices in equation (1) are mapped as

$$\begin{aligned} A_0 &= \begin{bmatrix} I_n - \Lambda W_n & 0 & 0 \\ -\lambda^p \widehat{W} & 1 & 0 \\ -\lambda^r \widehat{W} & -\delta & 1 \end{bmatrix}, A_1 = \begin{bmatrix} \Phi_1 + \Pi_1 W_n & \Gamma_1 & \Psi_1 \\ \pi_1^p \widehat{W} & \gamma_1^p & \psi_1^p \\ \pi_1^r \widehat{W} & \gamma_1^r & \psi_1^r \end{bmatrix}, \\ A_2 &= \begin{bmatrix} \Phi_2 + \Pi_2 W_n & \Gamma_2 & \Psi_2 \\ \pi_2^p \widehat{W} & \gamma_2^p & \psi_2^p \\ \pi_2^r \widehat{W} & \gamma_2^r & \psi_2^r \end{bmatrix}, \text{ and } M = \begin{bmatrix} \mu_n \\ \mu^p \\ \mu^r \end{bmatrix}. \end{aligned}$$

Equation (8) is an equation system consisting of 3 structural regressions. The dynamics of state-level income growth is explained in the first regression, where Φ_1 and Φ_2 are the unknown parameter matrices denoting the auto-relationship with time lags, Γ_1 and Γ_2 measure the impact of lagged inflations, Ψ_1 and Ψ_2 denote the impact of lagged federal funds rates, and μ_n measures the state-specific fixed effects. We also introduce parameter matrices of Π_1 and Π_2 in addition to Λ in the first regression, which allows state-level income growths to depend on time lagged neighborhood effects as well as contemporaneous neighborhood effects. The second regression describes the moves of inflation rate as a function of the weighted averages of both contemporaneous and subsequent state incomes, whose effects are measured using λ^p, π_1^p, and π_2^p; the lagged inflations, whose effects are measured by γ_1^p and γ_2^p; the lagged federal funds rates, whose effects are measured by ψ_1^p and ψ_2^p; and a constant term with parameter of μ^p. The last regression describes the changes of federal funds rate as a function of both contemporaneous and subsequent state incomes, whose effects are measured using λ^r, π_1^r, and π_2^r; the contemporaneous and lagged inflations, whose effects are measured by δ, γ_1^r, and γ_2^r; the lagged federal funds rates, whose effects are measured by ψ_1^r and ψ_2^r; and a constant term with parameter of μ^r.

Since neither p_t nor r_t affects $y_{it}s$ contemporaneously, we can split the structure of equation (8) and estimate the equation system of $y_{it}s$ independently from the equation system of p_t and r_t without loss of efficiency. The estimation procedure of each system is illustrated separately in the next two sub-sections.

4.2. State-Level Equation System and Estimation

The first regression of equation (8), which describes the state-level income dynamics has the following form,

$$\begin{aligned} y_{nt} &= \Lambda W_n y_{nt} + \Phi_1 y_{nt-1} + \Phi_2 y_{nt-2} + \Pi_1 W_n y_{nt-1} + \Pi_2 W_n y_{nt-2} \\ &\quad + \Gamma_1 p_{t-1} + \Psi_1 r_{t-1} + \Gamma_2 p_{t-2} + \Psi_2 r_{t-2} + \mu_n I_n + u_{nt}, \end{aligned}$$

where each of (Γ_1, Ψ_1, Γ_2, Ψ_2, and μ_n) is a parameter vector with n unknown elements to be estimated.[14] The number of unknown parameters in the parameter matrices of (Λ, Φ_1, Φ_2, Π_1, and Π_2) depends on the restrictions we put on states' income reaction pattern. By allowing heterogeneity across regions, we specify each of these parameter matrices as a diagonal square matrix with a dimension of n, whose diagonal elements have the values of 8 unknown coefficients: the values of the first m_1 diagonal elements are set equal; the values of the second m_2 diagonal elements are set equal, $\cdots$, and the last m_8 diagonal elements are set equal.

Thus far, the number of parameters in A_0^{11} that need to be estimated has reduced to 8. The block-diagonal feature of A_0^{11} allows us to write the equation system of $y_{it}s$ in 8 separate regressions. To avoid complex subscriptions, we suppress parameters' subscript for region, and simply let m indicate the number of states in a given region, and y_{mt} denote an $m \times 1$ vector consisting of y_{it}s ($i \in [1, 48]$) in a typical region. The vector form of the economic system describing the dynamic feedbacks of state income growth in a given region can be written as

$$\begin{aligned} y_{mt} &= \lambda W_m y_{mt} + \phi_1 y_{mt-1} + \phi_2 y_{mt-2} + \pi_1 W_m y_{mt-1} + \pi_2 W_m y_{mt-2} \\ &\quad + \gamma_1 p_{t-1} + \psi_1 r_{t-1} + \gamma_2 p_{t-2} + \psi_2 r_{t-2} + \mu I_m + u_{mt}, \end{aligned} \tag{9}$$

[14] Since Γ_1, Ψ_1, Γ_2, and Ψ_2 denote state's response to the changes of price level and federal funds rates, by allowing states to have heterogenous responses, these parameters will have an $n \times 1$ structure. Parameter μ_n measures state-specific fixed effects, and therefore, it has n elements to be estimated.

where W_m is an $m \times m$ weight matrix, y_{mt-j} $(j = 0, 1, 2) = \begin{bmatrix} y_{1t-j} & y_{2t-j} & \cdots \end{bmatrix}'$ is an $m \times 1$ vector of state-level income growth rate at time $t-j$, r_{t-j} $(j = 1, 2)$ and p_{t-j} $(j = 1, 2)$ are national-level nominal effective funds rate, I_m is a $m \times m$ identity matrix indicating state individual fixed effect, and inflation rate at time $t - j$, $u_{mt} = [u_{1t}, u_{2t}, \cdots]'$ is the vector of iid distributed disturbance terms, $(\lambda, \phi_1, \phi_2, \pi_1, \pi_2, \gamma_1, \psi_1, \gamma_2, \psi_2, \mu)$ are unknown parameters to be estimated, in which $(\lambda, \phi_1, \phi_2, \pi_1, \pi_2)$ are scalars and $(\gamma_1,$ $\psi_1,$ $\gamma_2,$ $\psi_2,$ $\mu)$ are all $m \times 1$ vector coefficients. The weight matrix W_m is then row-normalized to insure the existence of λ.[15] The set of parameters needs to be repeatedly estimated for each region.

Stacking the observations over T time periods together, we can rewrite equation(9) as,

$$\begin{aligned} y_{mT} &= \lambda(I_T \otimes W_m)y_{mT} + \phi_1 y_{-1,mT} + \phi_2 y_{-2,mT} + \pi_1(I_T \otimes W_m)y_{-1,mT} \\ &\quad + \pi_2(I_T \otimes W_m)y_{-2,mT} + (p_{-1,T} \otimes I_m)\gamma_1 + (r_{-1,T} \otimes I_m)\psi_1 \\ &\quad + (p_{-2,T} \otimes I_m)\gamma_2 + (r_{-2,T} \otimes I_m)\psi_2 + \mu(l_T \otimes I_m) + u_{mT}, \end{aligned} \tag{10}$$

where

$$\begin{aligned} y_{mT} &= [y'_{m1}, y'_{m3}, \cdots, y'_{mT}]', \\ y_{-1,mT} &= [y'_{m0}, y'_{m1}, \cdots, y'_{mT-1}]', \\ y_{-2,mT} &= [y'_{m-1}, y'_{m0}, \cdots, y'_{mT-2}]', \\ \text{and } u_{mT} &= [u'_{m1}, u'_{m3}, \cdots, u'_{mT}]', \end{aligned}$$

denoting $mT \times 1$ vectors. l_T is a $T \times 1$ vector of ones;

$$\begin{aligned} r_{-1,T} &= [r_0, r_1, \cdots, r_{T-1}]', \\ r_{-2,T} &= [r_{-1}, r_0, \cdots, r_{T-2}]', \\ p_{-1,T} &= [p_0, p_1, \cdots, p_{T-1}]', \\ \text{and } p_{-2,T} &= [p_{-1}, p_0, \cdots, p_{T-2}]', \end{aligned}$$

denoting $T \times 1$ vectors.

For simplicity, let $M = mT$, $W_M = (I_T \otimes W_m)$,[16] $C_M = (l_T \otimes I_m)$, $p_{-1,M} = p_{-1,T} \otimes I_m$, $r_{-1,M} = r_{-1,T} \otimes I_m$, $p_{-2,M} = p_{-2,T} \otimes I_m$, and $r_{-2,M} = r_{-2,T} \otimes I_m$, we have

$$\begin{aligned} y_M &= \lambda W_M y_M + \phi_1 y_{-1,M} + \phi_2 y_{-2,M} + \pi_1 W_M y_{-1,M} + \pi_2 W_M y_{-2,M} \\ &\quad + p_{-1,M}\gamma_1 + r_{-1,M}\psi_1 + p_{-2,M}\gamma_2 + r_{-2,M}\psi_2 + \mu C_M + u_M. \end{aligned} \tag{11}$$

or

$$y_M = \lambda W_M y_M + Z_M \rho + u_M, \tag{12}$$

[15] The row-normalization condition of the weight matrices is imposed to enforce the spatial weight matrix, W_m, and corresponding $(I_m - \lambda W_m)$ to be uniformly bounded in both row and column sums in absolute value. See Horn and Johnson (1985) for the properties of those matrix norms. When W_m is row normalized, $\|W_m\| \leq 1$. For $(I_m - \lambda W_m)$ to be nonsingular, the sufficient condition is that $|\lambda| < 1$. Because this condition is not necessary, the absolute values of the estimated λ does not have to be less than one.

[16] Because W_m is a zero diagonal matrix, the diagonal elements of W_M are zeros.

where

$$Z_M = [y_{-1,M}, y_{-2,M}, W_M y_{-1,M}, W_M y_{-2,M}, p_{-1,M}, r_{-1,M}, p_{-2,M}, r_{-2,M}, C_M],$$

denoting a $M \times (4+5m)$ matrix, and the coefficient vector ρ has $4+5m$ elements as

$$\rho = \left[\begin{array}{ccccccccc} \phi_1 & \phi_2 & \pi_1 & \pi_2 & \gamma_1' & \psi_1' & \gamma_2' & \psi_2' & \mu' \end{array} \right]'.$$

Before proceeding to estimate equation (11), we want to test whether it is necessary to include the contemporaneous spatial effects term, $\lambda W_M y_M$, in the model. Suppose the assumption of no contemporaneous interactions between states ($\lambda = 0$) made in previous studies is legitimate, after taking account of $W_M y_{-1,M}$ and $W_M y_{-2,M}$ in the model, there should not be any spatial correlation left in the error terms. Mathematically, we describe this using a restricted model as one of our benchmark models,

$$\begin{aligned} y_M \quad = \quad & \phi_1 y_{-1,M} + \phi_2 y_{-2,M} + \pi_1 W_M y_{-1,M} + \pi_2 W_M y_{-2,M} \\ & + p_{-1,M}\gamma_1 + r_{-1,M}\psi_1 + p_{-2,M}\gamma_2 + r_{-2,M}\psi_2 + \mu C_M + u_M, \end{aligned} \tag{13}$$

in which the fitted residuals of u_M are expected to display no spatial correlations. The specification test is carried using Moran's I test, which is the most commonly used specification test for spatial autocorrelation developed by Moran (1950), and was generalized to regression residuals by Cliff and Ord (1972). The notation of Moran's I statistic for equation (13) is $I = e'W_M e/e'e$, where e is the vector of OLS residuals and W_M is a row-standardized weight matrix. Inference for Moran's I is based on a normal approximation, using a standardized z-value obtained from expressions for the mean and variance of the I statistic as $[I - Mean(I)]/var(I)$. If the standardized statistic is greater than 2.58, we reject the null of no spatial correlation at 1% significance level. Table 2 reports the standardized Moran's statistics for the 8 regions, which indicate strong spatial correlations residing in the fitted residuals, and the null hypothesis is rejected in all the regions.

Table 2: Standardized Moran's I statistics

Regions	New England	Mid -east	Great Lakes	Plains	South -east	South -west	Rocky Mountain	Far West
Moran's I	26.75	20.36	23.65	16.19	42.11	10.09	10.66	14.67

Based on the test results, our model accounting for contemporaneous spatial effects are well motivated. Previous studies, such as Carlina and Defina (1999) have estimated models that allow for some degree of dependency between income growth rates, but only with a lag. In these studies, the identification approach the authors employ requires them assume that income growth rates for states or regions do not contemporaneously impact other states and regions. This assumption implies that a state's immediate impulse response to a monetary policy shock is governed entirely by the sensitivity of the state's income to the monetary target, typically the Federal Funds rate target. The implication suggests that states with little sensitivity to interest rates will not be influenced quickly by a monetary policy shock.

Equation (12) has the functional form of a spatial autoregressive (SAR) model with one spatial lag of $\lambda W_M y_M$. The total number of parameters that need to be estimated is $4+5m$. We employ the Maximum Likelihood Estimator (MLE) to estimate the equation and present the MLEs in this section.

From equation(12), if u_M is normally distributed as $N(0, \sigma^2 I_M)$, the log likelihood function of this model is

$$\ln L_M = -\frac{M}{2}\ln(2\pi) - \frac{M}{2}\ln(\sigma^2) + \ln|I_M - \lambda W_M| - \frac{1}{2\sigma^2}u'_M(\theta)u_M(\theta),$$

with the first order derivatives being

$$\begin{aligned}\frac{\partial L_M}{\partial \lambda} &= \frac{\partial \ln|I_M - \lambda W_M|}{\partial \lambda} - \frac{1}{2\sigma^2}\frac{\partial(u'_M(\theta)u_M(\theta))}{\partial \lambda} \\ &= -tr(G_M(\lambda)) + \frac{1}{\sigma^2}(G_M(\lambda)Z_M\rho)'u_M(\theta) + \frac{1}{\sigma^2}u'_M(\theta)G_M(\lambda)u_M(\theta),\end{aligned}$$

where

$$G_M(\lambda) = W_M(I_M - \lambda W_M)^{-1},$$

and

$$\begin{aligned}\frac{\partial L_M}{\partial \rho} &= \frac{1}{\sigma^2}Z'_M u_M(\theta), \\ \frac{\partial L_M}{\partial \sigma^2} &= -\frac{M}{2\sigma^2} + \frac{1}{2\sigma^4}u'_M(\theta)u_M(\theta).\end{aligned}$$

4.3. National-Level Equation System and Estimation

The last two regressions of equation (8), which describe the behaviors of inflation rate and federal funds rate have a form of

$$p_t = \lambda^p \widehat{W} y_{nt} + \pi_1^p \widehat{W} y_{nt-1} + \gamma_1^p p_{t-1} + \psi_1^p r_{t-1} + \pi_2^p \widehat{W} y_{nt-2} + \gamma_2^p p_{t-2} + \psi_2^p r_{t-2} + \mu^p + u_{pt}, \quad (14)$$

and

$$r_t = \delta p_t + \lambda^r \widehat{W} y_{nt} + \pi_1^r \widehat{W} y_{nt-1} + \gamma_1^r p_{t-1} + \psi_1^r r_{t-1} + \pi_2^r \widehat{W} y_{nt-2} + \gamma_2^r p_{t-2} + \psi_2^r r_{t-2} + \mu^r + u_{rt}, \quad (15)$$

where $\widehat{W}$ is a $1 \times n$ weight vector averaging the state-level incomes,[17] ys are vectors of state-level income growth rates, ps are CPIs, rs are federal funds rates, us are disturbance terms, (δ, λs, πs, γs, ψs, and μs) are all scalar coefficients.

Let $\overline{y}_t = \widehat{W} y_{nt}$, $\overline{y}_{t-1} = \widehat{W} y_{nt-1}$, and assume the state-level incomes can be observed before p_t and r_t are determined,[18] the equation system is written as the following vector form

[17] The construction of $\widehat{W}$ is different from that of W_n. Instead of based on adjacency, the income growth rate, y_{nt}, in the national-level equation system is weighted based on the size of the economy in each state. Therefore the produce of $\widehat{W}$ and y_{nt} results in the average state income growth rate.

[18] Our estimation here is based on quarterly date. The Fed have monthly data on income levels before they decide on the monetary policy actions. Thus, the assumption of predetermined p_t and r_t can be justified.

$$\begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} p_t \\ r_t \end{bmatrix} = \begin{bmatrix} \mu^p \\ \mu^r \end{bmatrix} + \begin{bmatrix} \lambda^p & \pi_1^p & \pi_2^p \\ \lambda^r & \pi_1^r & \pi_2^r \end{bmatrix} \begin{bmatrix} \overline{y}_t \\ \overline{y}_{t-1} \\ \overline{y}_{t-2} \end{bmatrix} + \begin{bmatrix} \gamma_1^p & \psi_1^p \\ \gamma_1^r & \psi_1^r \end{bmatrix} \begin{bmatrix} p_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \gamma_2^p & \psi_2^p \\ \gamma_2^r & \psi_2^r \end{bmatrix} \begin{bmatrix} p_{t-2} \\ r_{t-2} \end{bmatrix} + \begin{bmatrix} u_{pt} \\ u_{rt} \end{bmatrix}. \tag{16}$$

Because of the endogeneity of Equation (16), OLS estimates will be biased. We rewrite it in a reduced form that can be directly estimated as

$$\begin{bmatrix} p_t \\ r_t \end{bmatrix} = A \begin{bmatrix} \overline{y}_t \\ \overline{y}_{t-1} \\ \overline{y}_{t-2} \end{bmatrix} + B \begin{bmatrix} p_{t-1} \\ r_{t-1} \end{bmatrix} + C \begin{bmatrix} p_{t-2} \\ r_{t-2} \end{bmatrix} + D + \begin{bmatrix} e_{pt} \\ e_{rt} \end{bmatrix}, \tag{17}$$

where

$$\begin{aligned} A &= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}^{-1} \begin{bmatrix} \lambda^p & \pi_1^p & \pi_2^p \\ \lambda^r & \pi_1^r & \pi_2^r \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} \lambda^p & \pi_1^p & \pi_2^p \\ \lambda^r & \pi_1^r & \pi_2^r \end{bmatrix} \\ &= \begin{bmatrix} \lambda^p & \pi_1^p & \pi_2^p \\ \delta\lambda^p + \lambda^r & \delta\pi_1^p + \pi_1^r & \delta\pi_2^p + \pi_2^r \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} B &= \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}^{-1} \begin{bmatrix} \gamma_1^p & \psi_1^p \\ \gamma_1^r & \psi_1^r \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} \gamma_1^p & \psi_1^p \\ \gamma_1^r & \psi_1^r \end{bmatrix} \\ &= \begin{bmatrix} \gamma_1^p & \psi_1^p \\ \delta\gamma_1^p + \gamma_1^r & \delta\psi_1^p + \psi_1^r \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} C &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}^{-1} \begin{bmatrix} \gamma_2^p & \psi_2^p \\ \gamma_2^r & \psi_2^r \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} \gamma_2^p & \psi_2^p \\ \gamma_2^r & \psi_2^r \end{bmatrix} \\ &= \begin{bmatrix} \gamma_2^p & \psi_2^p \\ \delta\gamma_2^p + \gamma_2^r & \delta\psi_2^p + \psi_2^r \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} D &= \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mu^p \\ \mu^r \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} \mu^p \\ \mu^r \end{bmatrix} \\ &= \begin{bmatrix} \mu^p \\ \delta\mu^p + \mu^r \end{bmatrix}, \end{aligned}$$

and

$$\begin{bmatrix} e_{pt} \\ e_{rt} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_{pt} \\ u_{rt} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} u_{pt} \\ u_{rt} \end{bmatrix} = \begin{bmatrix} u_{pt} \\ \delta u_{pt} + u_{rt} \end{bmatrix}.$$

By identifying the elements in matrices of A, B, C,and D as well as the variance-covariance matrix of residuals es, we can calibrate for the coefficients in the structural form of equation (16). The technical details of the calibration is provided as follows.

After rewriting the vector equation (17) in two separate functions as

$$p_t = A_{11}\overline{y}_t + A_{12}\overline{y}_{t-1} + A_{13}\overline{y}_{t-2} + B_{11}p_{t-1} + B_{12}r_{t-1} + C_{11}p_{t-2} + C_{12}r_{t-2} + D_1 + e_{pt}, \quad (18)$$

and

$$r_t = A_{21}\overline{y}_t + A_{22}\overline{y}_{t-1} + A_{23}\overline{y}_{t-2} + B_{21}p_{t-1} + B_{22}r_{t-1} + C_{21}p_{t-2} + C_{22}r_{t-2} + D_2 + e_{rt}, \quad (19)$$

the following equalities hold

$$\begin{aligned} A_{11} &= \lambda^p, A_{12} = \pi_1^p, A_{13} = \pi_2^p, \\ B_{11} &= \gamma_1^p, B_{12} = \psi_1^p, \\ C_{11} &= \gamma_2^p, C_{12} = \psi_2^p, \\ D_1 &= \mu^p, \end{aligned}$$

and

$$\begin{aligned} A_{21} &= \delta\lambda^p + \lambda^r, A_{22} = \delta\pi_1^p + \pi_1^r, A_{23} = \delta\pi_2^p + \pi_2^r, \\ B_{21} &= \delta\gamma_1^p + \gamma_1^r, B_{22} = \delta\psi_1^p + \psi_1^r, \\ C_{21} &= \delta\gamma_2^p + \gamma_2^r, C_{22} = \delta\psi_2^p + \psi_2^r, \\ D_2 &= \delta\mu^p + \mu^r. \end{aligned}$$

Let $\Omega = E\left(e_t e_t' | \begin{matrix} \overline{y}_t \\ \overline{y}_{t-1} \\ \overline{y}_{t-2} \end{matrix}, \begin{matrix} p_{t-1} \\ r_{t-1} \end{matrix}, \begin{matrix} p_{t-2} \\ r_{t-2} \end{matrix} \right)$, where $e_t = \begin{bmatrix} e_{pt} \\ e_{rt} \end{bmatrix}$, denote the variance-covariance matrix of the disturbance terms in equation (17), and use OLS to estimate the elements of A, B, C, D and Ω. Then λ^p, π_1^p γ_1^p, ψ_1^p, π_2^p, γ_2^p and ψ_2^p in equation (14) can be identified through the estimates of A_{11}, A_{12}, A_{13}, B_{11}, B_{12}, C_{11}, C_{12}, and D_1 in equation (18). To identify δ, we assume the structural shocks in equation (16) are uncorrelated as

$$E\left(u_t u_t' | \begin{matrix} \overline{y}_t \\ \overline{y}_{t-1} \\ \overline{y}_{t-2} \end{matrix}, \begin{matrix} p_{t-1} \\ r_{t-1} \end{matrix}, \begin{matrix} p_{t-2} \\ r_{t-2} \end{matrix} \right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix},$$

where $u_t = \begin{bmatrix} u_{pt} \\ u_{rt} \end{bmatrix}$. The independence of structural shocks also suggests

$$
\begin{aligned}
\Omega &= E\left(e_t e_t' | \begin{array}{c}\overline{y}_t \\ \overline{y}_{t-1} \\ \overline{y}_{t-2}\end{array}, \begin{array}{c}p_{t-1} \\ r_{t-1}\end{array}, \begin{array}{c}p_{t-2} \\ r_{t-2}\end{array}\right) \\
&= \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}^{-1} E\left(u_t u_t' | \begin{array}{c}\overline{y}_t \\ \overline{y}_{t-1} \\ \overline{y}_{t-2}\end{array}, \begin{array}{c}p_{t-1} \\ r_{t-1}\end{array}, \begin{array}{c}p_{t-2} \\ r_{t-2}\end{array}\right) \left(\begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}^{-1}\right)' \\
&= \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} E\left(u_t u_t' | \begin{array}{c}\overline{y}_t \\ \overline{y}_{t-1} \\ \overline{y}_{t-2}\end{array}, \begin{array}{c}p_{t-1} \\ r_{t-1}\end{array}, \begin{array}{c}p_{t-2} \\ r_{t-2}\end{array}\right) \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} a & a\delta \\ a\delta & a\delta^2 + b \end{bmatrix}.
\end{aligned}
$$

An estimate of δ can be obtained using the ratio of the estimated $a\delta$ and a in Ω. Given the estimated δ and the set of the estimated structural parameters in equation (14), we can identify the rest of the structural parameters in equation (15) through those equalities.

5. ESTIMATION RESULTS

5.1. Discussions of State-level Regression

This section reviews our main results of the state income dynamics displayed in equation (9). First we present the estimated results of our income dynamic model, than we carry out three hypotheses to demonstrate the robustness of our spatial income model.

Table 3 presents the ML parameter estimates of λ, ϕ_1, ϕ_2, π_1, and π_2 of equation (9) for each region separately. The contemporaneous neighborhood effect coefficients, λs, are observed to play an important role in amplifying the responses of regional economies. The t statistics for all λs are in the rejection region at 1% significance level. The values of λs range between 0.40 and 0.71, which translates into an additional magnification effect transmitted from neighboring states. In an extreme case, suppose state i is not sensitive to monetary policy at all, the direct response of state i's economy to a policy shock is zero. But state i's total impulse response to a policy shock is not zero, instead, it should be the product of state i's neighborhood coefficient λ and the weighted average of responses from its neighboring states in the region. As long as state i's neighboring states are sensitive to a shock, its economy can still be influenced quite a lot by the policy change, indirectly through the interaction with other states in the same region. The existence of contemporaneous neighborhood effects will surely alter the picture of aggregate response of the whole national economy, which serves as the target of the Fed's policy. Our findings here can add insight to the decisions made by policymakers and strengthen the efficacy of their policy actions. The parameter estimates of one period lagged neighborhood effects, π_1 are still significant for 7 out of 8 regions, but with a much smaller value ranging from 0.12 to 0.34. The significance of the two periods lagged neighborhood effects, π_2 diminishes quickly for most of the regions. The fall in the magnitude and significance level of the lagged neighborhood effects again proves that a more important role in the income dynamic model is played by the contemporaneous interactions but not the delayed ones.

The estimates of ϕ_1 are negative and in similar scales of the values of π_1; while the estimates of ϕ_2 are only significant for 3 regions with various signs. The coefficient estimates of lagged ys were usually found to be positive in previous studies. The negative sign of ϕ_1s and ϕ_2s in this paper may result from the newly introduced neighborhood effects, λ and π_1.[19] As the influence of the contemporaneous neighborhood effects, λ,is overwhelming over the impacts from all the rest sources, omitting the simultaneous movements in states' responses in the model specification might bias the estimates and misinterpret the impulse response function.

Table 3: Estimation Results of y_M Regressions

	Estimates (t-statistics is in the bottom)							
	New England	Mid -east	Great Lakes	Plains	South -east	South -west	Rocky Mountain	Far West
λ	0.69***	0.62***	0.69***	0.49***	0.71***	0.41***	0.40***	0.54***
	35.29	15.91	34.88	14.31	27.95	10.46	4.57	16.96
ϕ_1	-0.07*	-	-	-	-	-0.07	-	-0.03
		0.36***	0.14***	0.17***	0.14***		0.13***	
	-1.84	-7.88	-3.14	-4.70	-5.02	-1.54	-2.840	-0.62
ϕ_2	0.04	-0.05	-0.05	-	0.04	0.09*	-0.06	0.12**
				0.14***				
	1.00	-1.08	-1.17	-3.94	1.37	1.90	-1.33	2.54
π_1	0.12**	0.34***	0.14***	0.13**	0.15***	0.14**	0.07	0.12**
	2.55	6.47	2.79	2.47	3.90	2.19	1.06	1.97
π_2	0.02	0.12**	0.09*	0.12**	-0.03	-0.01	0.03	-0.04
	0.33	2.29	1.77	2.35	-0.90	-0.18	0.54	-0.72

***Indicates significance level at 1% or less (p-value is 2.58); **Indicates significance level at 5% or less (p-value is 1.96); *Indicates significance level at 10% or less (p-value is 1.65).

To find out how robust our spatial income dynamic model of equation (10) is, we would like to compare it with several competing alternatives and carry the following hypothesis tests.

5.1.1. Hypothesis test 1 - Joint significance of elements in each parameter vector of (γ_1, ψ_1, γ_2, ψ_2, and δ)

In addition to the state-varying fixed individual effects, both inflation rate and federal funds rate are assumed to have various marginal impacts on states.[20] Given that the total number of observation time periods is 105, and the total number of states is 48, it is not surprising to see most of the estimated parameters in these parameter vectors of (γ_1, ψ_1, γ_2, ψ_2, and δ) are not significant.

To find out whether the elements within each of the parameter vectors in front of CPIs and fed funds are jointly significant, we carry out four likelihood ratio tests separately for each region. The unrestricted model for these likelihood ratio tests is our spatial equation(10), and the alternative models restrict one of the parameter vectors

[19] The OLS estimates of equation (13) - the restricted version without comtemporaneous effects - have very similar results compared with Table 3: μ_1s are most significant and negative; β_{11}s are most significant and positive; μ_2 and γ_{11} are most not significant.

[20] Recall β_{12}, β_{13}, γ_{12}, γ_{13}, and β_{10} are all $m \times 1$ parameter vectors instead of scalars (where m is the number of states in a given region). β_{12}/γ_{12} measures the impact of CPI growth rate with one time lag/two time lags on state income growth rate, β_{13}/γ_{13} measures the impact of federal funds rate with one time lag/two time lags on state income growth rate, and β_{10} measures the state individual fixed effects.

of (γ_1, ψ_1, γ_2, ψ_2) to be zero. The test results are reported in **Table 4**. The likelihood ratio test statistics follow a $\chi^2(m)$ distribution. Recall the number of states in each region differs, the critical values of a $\chi^2(m)$ distribution at a given significance level should vary across regions. The parameter vectors of the one time period lagged CPI rate and federal funds rate, γ_1 and ψ_1, are jointly significant for most of the regions; while the parameter vectors of the two time period lagged ones, γ_2 and ψ_2, are jointly significant only for some of the regions. For comparison reason, although the parameter vectors of (γ_1, ψ_1, γ_2, ψ_2) are not always jointly significant for every region, we still keep all of them in our estimation routine.

Table 4: Likelihood Ratio Tests of Joint Significance

Regions	$\chi^2(m)$ statistics Test for γ_1 (p_{-1})	 Test for ψ_1 (r_{-1})	 Test for γ_2 (p_{-2})	 Test for ψ_2 (r_{-2})
New England ($m = 6$)	12.28*	18.22***	7.80	16.76**
Mideast ($m = 5$)	14.17**	8.72	1.44	9.18
Great Lakes ($m = 5$)	13.16**	5.51	0.98	8.97
Plains ($m = 7$)	14.04*	30.54***	4.23	14.55**
Southeast ($m = 12$)	20.86*	41.63***	20.98*	28.08***
Southwest ($m = 4$)	2.80	9.44*	19.50***	11.88**
Rocky Mountain ($m = 5$)	41.45***	7.28	33.26***	13.82**
Far West ($m = 4$)	6.80	8.81*	7.29	7.40

***Indicates significance level at 1% or less; **Indicates significance level at 5% or less ; *Indicates significance level at 10% or less. m is the degree of freedom, representing the number of states in a given region. Since different regions have different number of states, the critical value varies over regions.

5.1.2. Hypothesis test 2 - Uniform elements in each parameter vector of (γ_1, ψ_1, γ_2, ψ_2, *and* δ)

Our unrestricted spatial dynamic model of equation(10) allows inflation rate and federal funds rate to have various marginal effects over states, which implies that the sensitivity of state economy to the changes of aggregate price level and monetary policy varies over the borders. A number of existing studies have shared the same view arguing for the heterogeneity in state level income dynamics in response to monetary policy. To check for the legitimacy of this argument, we carry our second hypothesis test here to find out whether the elements in each parameter vector of (γ_1, ψ_1, γ_2, ψ_2, and δ) are uniform. The alternative model for testing the uniform elements in γ_1 has the same formula as the unrestricted one, except for replacing the Kronecker tensor product of $(p_{-1,T} \otimes I_m)$ in equation(10) to $(p_{-1,T} \otimes l_m)$. The estimated parameter of γ_1 in the restricted model is then a scalar instead of a vector. The alternative models for testing the uniformity of ψ_1, γ_2, ψ_2, and δ can be generated in a similar way.

Table 5 reports the likelihood ratios test results of the unrestricted model and the alternative. The likelihood ratio test statistics follow a $\chi^2(m-1)$ distribution, whose degrees of freedom is based on the number of states in each given region. Column 2 (ψ_1) and 4 (ψ_2) of **Table 5** denote the test results of parameter vectors in front of monetary policy change. There are 5 out of 8 regions, who reject the null hypothesis at a 5% significance level for the test of ψ_1, while there are only 3 out of 8 regions rejecting the null hypothesis for the test of ψ_2. These findings suggest that the immediate responses of states' economy to monetary policy shock tend to be heterogenous over the border, but they will eventually converge to a similar level over time. We believe the quick die-out of the heterogenous reactions of states attributes to the existence of contemporaneous interactions among states' economy - the neighborhood effects, which helps to promote communication across states and expedites the transmission of a monetary policy's impact in the nation. For the regions that can not reject the hypothesis of uniform influence of one period lagged monetary policy shock (ψ_1), such as Mideast, Great Lakes, and Rocky Mountain, we conjecture that the information exchange process between states was completed immediately through the contemporaneous interactions within a region, which conceals the possible heterogeneous sensitivity of states to the changes of monetary policy and leads to a uniform response across states. The implication of these test results is profound: in the regions where neighborhood effects can spread out in a short time period, it will be safe to assume homogeneous response to monetary policy shock over states, even states have actually heterogenous sensitivities to the shock. Because of the quick interactions between states, the heterogeneity in sensitivity can be smoothed out and will not matter any more to the evaluation of aggregate national income change.

Most of the regions (6 out of 8) can not reject the null hypothesis of uniform response to aggregate price change. This is true for both one period lagged and two periods lagged inflation rates (γ_1 and γ_2). The advanced modern communications networks and the wide scale of internet shops remove the locational restrictions that consumers face and integrate local markets into a national wide one. It is not surprising to find states respond to the aggregate price level uniformly.

Table 5: Likelihood Ratio Tests of Uniform Elements in (γ_1, ψ_1, γ_2, ψ_2)

Regions	$\chi^2(m-1)$ statistics Test for γ_1 (p_{-1})	Test for ψ_1 (r_{-1})	Test for γ_2 (p_{-2})	Test for ψ_2 (r_{-2})
New England ($m=6$)	5.06	12.46**	7.58	12.60**
Mideast ($m=5$)	6.81	1.13	3.27	3.90
Great Lakes ($m=5$)	8.17	4.85	1.33	1.94
Plains ($m=7$)	9.52	28.54***	1.67	13.09**
Southeast ($m=12$)	21.00**	35.67***	17.06	16.86
Southwest ($m=4$)	0.59	9.10**	8.69**	8.62**
Rocky Mountain ($m=5$)	38.09***	5.54	17.85***	3.36
Far West ($m=4$)	3.91	8.27**	1.36	4.20

***Indicates significance level at 1% or less; **Indicates significance level at 5% or less ; *Indicates significance level at 10% or less. $m-1$ is the degree of freedom, where m represents the number of states in a given region. Since different regions have different number of states, the critical value varies over regions.

In addition to separate uniform tests regarding each of the parameter vector of (ϕ_1, π_1, ϕ_2, and π_2), we also report another uniform test, which tests the uniformity of these vectors jointly. We describe the alternative model as,

$$\begin{aligned} y_{mT} &= \lambda(I_T \otimes W_m)y_{mT} + \phi_1 y_{-1,mT} + \phi_2 y_{-2,mT} + \pi_1(I_T \otimes W_m)y_{-1,mT} \\ &\quad +\pi_2(I_T \otimes W_m)y_{-2,mT} + (p_{-1,T} \otimes l_m)\gamma_1 + (r_{-1,T} \otimes l_m)\psi_1 \\ &\quad +(p_{-2,T} \otimes l_m)\gamma_2 + (r_{-2,T} \otimes l_m)\psi_2 + \delta(l_T \otimes I_m) + u_{mT}, \end{aligned} \tag{20}$$

where γ_1, ψ_1, γ_2, and ψ_2 all become scalar parameters for a given region. We first present the estimated results of the alternative model in **Table 6,** then report the test statistics in **Table 7**.

Table 6: Estimation Results of Equation (20)

	Estimates (t-statistics is in the bottom)							
	New England	Mid -east	Great Lakes	Plains	South -east	South -west	Rocky Mountain	Far West
λ	0.68***	0.68***	0.67***	0.48***	0.69***	0.38***	0.37***	0.51***
	34.18	15.13	22.10	7.75	34.10	9.20	8.28	15.39
ϕ_1	-0.07*	-0.31***	-0.11**	-0.17***	-0.07**	-0.02	-0.07	0.04
	-1.79	-6.99	-2.43	-4.87	-2.47	-0.45	-1.49	0.72
ϕ_2	0.05	-0.01	0.00	-0.11***	0.10***	0.11**	0.00	0.19***
	1.23	-0.18	0.08	-3.28	3.72	2.34	-0.03	3.83
π_1	0.12**	0.30***	0.11**	0.14**	0.08**	0.09	0.01	0.06
	2.51	5.67	2.14	2.54	1.97	1.38	0.09	0.93
π_2	0.01	0.08	0.03	0.10*	-0.10***	-0.03	-0.02	-0.10*
	0.15	1.51	0.68	1.88	-2.60	-0.49	-0.39	-1.72
γ_1	-0.13**	-0.16***	-0.11**	-0.36**	-0.08*	-0.15	-0.23*	-0.14**
	-2.37	-2.64	-2.00	-2.37	-1.77	-1.53	-1.69	-1.97
ψ_1	0.0004**	0.0005**	0.0002	0.0008	0.0003*	0.0002	0.0006	0.0001
	2.26	2.41	1.20	1.46	1.74	0.62	1.30	0.47
γ_2	0.07	0.06	0.06	0.25	0.08*	0.33***	0.50***	0.17**
	1.21	0.92	1.02	1.54	1.74	3.30	3.94	2.42
ψ_2	-	-	-	-0.0009	-	-	-	-
	0.0004**	0.0004**	0.0005**		0.0005***	0.0007*	0.0016***	0.0005*
	-2.19	-2.06	-2.39	-1.64	-2.78	-1.79	-3.19	-1.78

***Indicates significance level at 1% or less (p-value is 2.58); **Indicates significance level at 5% or less (p-value is 1.96); *Indicates significance level at 10% or less (p-value is 1.65).

The estimated λs of the restricted model are all significant and have similar values as compared with the unstricted model in **Table 3**. The one time period lagged neighborhood effect (ϕ_1) and self-influence (π_1) tend to be more significant than two time periods lagged ones (ϕ_2 and π_2). The magnitude and significance of the estimated (ϕ_1, π_1, ϕ_2, and π_2) in **Table 6** are comparable to those in **Table 3**. inflation rate has a significant and negative effect on state economy for most regions (7 out of 8) in the following up period (γ_1), but the effects die out quickly in the second lagged period (γ_2). The impact of federal funds rate is more significant in the second following period (ψ_2) than in the first period (γ_2), but the magnitude is a very small scale (<-0.01).

Table 7: Likelihood Ratio Tests of Joint Uniformity of $(\gamma_1,\ \psi_1,\ \gamma_2,\ \psi_2)$

Regions	$\chi^2(4\times m-4)$ statistics
New England ($m=6$)	24.93
Mideast ($m=5$)	28.14**
Great Lakes ($m=5$)	41.99***
Plains ($m=7$)	38.94**
Southeast ($m=12$)	141.57***
Southwest ($m=4$)	34.30***
Rocky Mountain ($m=5$)	66.08***
Far West ($m=4$)	35.38***

***Indicates significance level at 1% or less; **Indicates significance level at 5% or less; *Indicates significance level at 10% or less. $4\times m-4$ is the degree of freedom, where m represents the number of states in a given region. Since different regions have different number of states, the critical value varies over regions.

The test statistics follow a χ^2 distribution with a degree of freedom of $(4\times m-4)$, where m denotes the number of neighbors in a region. The null hypothesis of uniform elements in $(\gamma_1,\ \psi_1,\ \gamma_2,\ \psi_2)$ is rejected in all the regions except for the New England Region.

5.1.3. Hypothesis test 3 - Uniform neighborhood effect coefficients (λs) over regions

In the estimation of the unrestricted model of equation(10) for each region, we allow the contemporaneous neighborhood effect coefficient λs to differ over regions, which implies that regions react differently to their neighbor's influence. To test whether the neighborhood effects differ across region, we establish an alternative model restricting λs to be the same for every region. This alternative model is estimated using all the observations from all 8 regions, which can be written as following,

$$\begin{aligned} y_{nT} \quad = \quad & \lambda(I_T\otimes W_n)y_{nT}+\phi_1 y_{-1,nT}+\phi_2 y_{-2,nT}+\pi_1(I_T\otimes W_n)y_{-1,nT} \\ & +\pi_2(I_T\otimes W_n)y_{-2,nT}+(p_{-1,T}\otimes I_n)\gamma_1+(r_{-1,T}\otimes I_n)\psi_1 \\ & +(p_{-2,T}\otimes I_n)\gamma_2+(r_{-2,T}\otimes I_n)\psi_2+\delta(l_T\otimes I_n)+u_{nT}, \end{aligned} \tag{21}$$

where $y_{nT}=[y'_{n1},y'_{n2},\cdots,y'_{nT}]'$, and y_{nt}s $(t\in[1,T])$ denotes vectors with 48 elements representing the income growth rates of the 48 continguous U.S. state at time t; $(\lambda,\ \phi_1,\ \phi_2,\ \pi_1,\ \pi_2)$ are scalars, and $(\gamma_1,\ \psi_1,\ \gamma_2,\ \psi_2,\ \delta)$ are 48×1 parameter vectors. Equation (21) serves as the null hypothesis model against the unrestricted version of equation(10). **Table 8** reports the likelihood ratio test statistics. The log-likelihood ratio follows a $\chi^2(7)$ distribution. The null hypothesis is rejected at 1% significance level, confirming that the influence of neighborhood effects varies over regions. Given different patterns of neighborhood structures across regions, the test result is expected. We report partial estimation results of equation (21) in **Table 9**.

Table 8: Likelihood Ratio Test
(Uniform Neighborhood Effects over Regions)

Restricted Model:	
Log-likelihood Value (same λs)	17641.18
Restricted Models:	
Log-likelihood Value - New England	2503.76
Log-likelihood Value - Mideast	2106.34
Log-likelihood Value - Great Lakes	2157.39
Log-likelihood Value - Plains	2094.80
Log-likelihood Value - Southeast	4929.52
Log-likelihood Value - Southwest	1536.78
Log-likelihood Value - Rocky Mountain	1757.81
Log-likelihood Value - Far West	1672.66
$\chi^2(7)$ statistics (=-2×(restricted -Σ unrestricted))	2235.76***

***Indicates significance level at 1% or less.

Table 9: Estimation Results of Equation (21)

Parameters	Estimates	T-statistics
λ	0.53***	23.88
ϕ_1	-0.07	-1.03
ϕ_2	-0.35***	-4.57
π_1	-0.14	-1.57
π_2	-0.17***	-8.95

5.2. Impulse Response Functions and Effects of Monetary Policy Shocks

In this section we compute the model's implied impact coefficients to a monetary policy shock, and impulse response for the system implied by the estimated coefficients. For our experiment we consider the effects of a 1 standard deviation monetary contraction, represented by an unexpected standard deviation increase in the Federal Funds rate. Finally we compare our results with a benchmark model that does not allow for contemporaneous spatial correlation of income growth rates.

The average impulse response functions for each region show the same general pattern. There is an initial rise in income growth followed by a steep decline that peaks about 10 quarters after the policy shock. The net impact on the level of income is a permanent negative reduction, while the growth rate stabilizes back at along its trend. These average responses are similar to the estimated impulse response functions to monetary policy shocks from aggregated US data using the same identification approach.

Across states there is substantial variation in response, and even greater variation across regions. In the long run, these differences disappear as the economy stabilizes. For instance, Oregon shows the greatest response a 1.2% reduction, while Rhode Island shows the least response just 0.001% cumulative response to the monetary policy shock.

To illustrate the importance of including the contemporaneous spatial correlation captured by λs in our estimation equation, we compare our results to results from two benchmark models. In the first model, we force $\lambda = 0$ (equation (13)), that is we restrain all spatial correlation to only enter the model with a time lag. This model is quite similar to the models studied in previous work, such as Carlina and Defina (1998). In the second model we allow λ to be non-zero, but we constrain the coefficients on the national level variables to be equal across states. This model assumes that though spatial correlation exists, there is no important difference across states in their sensitivities to Federal Reserve Policy and short term interest rates.

5.2.1. Comparisons to Benchmark Model 1

Tables 10 and 11 compares the cumulative response (after 20 quarters) to a one standard deviation Federal Funds Rate Increase in the unrestricted model and the model where λs are restricted to be zero (equation (13)). These tables indicate that the responses estimated from the restricted model tend to underestimate the cumulative impact of the a monetary policy shock. This result is not surprising since the absence of contemporaneous spatial correlation will tend to downplay the impact coefficient of a monetary policy shock, since in the period after the shock, the only way monetary policy affect state income growth is through the interest rate estimated by γ_1. In the unrestricted model on the other hand, monetary policy can also indirectly influence a state's income response through the spillover effect from its neighbors, captured in λs. As we showed in the previous section, λs are statistically significant, and omitting it from the estimation is likely to bias estimated impulse response functions.

Table 10: Baseline Case

Region	Average Impulse Response	Coefficient of Variation	Max-Min
1	0.04	0.24	0.64
2	0.78	1.00	0.81
3	0.42	1.55	0.98
4	0.09	0.42	2.02
5	0.24	0.52	2.26
6	0.30	2.50	0.16
7	0.53	1.63	0.98
8	0.46	0.49	2.01
ALL	0.22	0.60	2.65

Table 11: $\lambda = 0$ Case

Region	Average Impulse Response	Coefficient of Variation	Max-Min
1	0.05	0.20	0.54
2	0.26	1.00	0.72
3	0.04	1.02	0.93
4	0.08	0.12	1.89
5	0.21	0.34	2.10
6	0.26	3.76	0.16
7	0.47	1.32	0.87
8	0.41	0.49	1.81
ALL	0.19	0.39	2.45

The variation within regions is smaller than the variation across regions. The Coefficient of variation across all regions is 1.47 in the baseline case, which exceeds the variation in all but region 6, the Southwest region. Similarly, for the case when there are not contemporaneous spatial effects, $\lambda = 0$, the coefficient of variation is 1.36, greater than all but the Southwest, region 6. The relatively high variation in Southwest region might result from the small number of states it includes (12).

5.2.2. Comparisons to Benchmark Model 2

We also compare our model to a restricted model where the responses to interest rates and inflation, γ_1, ψ_1, γ_2, and ψ_2, are uniform within regions. The results for the state level equations are presented in **Table 6** above. Below we produce results for the full dynamic model and summarize the key findings in **Table 12**. Unlike the other models there is no regional heterogeneity in the response, so each state within a region responds identically. The estimated cumulative impact after 10 quarters (0.10) is also only half of the average cumulative impacts estimated in the unrestricted model (0.22), which are consistent with our previous hypothesis test results. States respond to monetary policy shock heterogenously, which is as important as neighborhood effcts, and should be modeled in income dynamic models.

Table 12 : Same γ_1, ψ_1, γ_2, and ψ_2 within Each Region

Region	Cumulative Impulse Response	Coefficient of Variation
1	0.11	-
2	0.12	-
3	0.07	-
4	0.05	-
5	0.01	-
6	0.19	-
7	0.20	-
8	0.05	-
ALL	0.10	1.37

6. CONCLUSIONS

This paper highlights the importance of spillover effects for the impact of monetary policy on state level income growth. This study advances the literature by relaxing the assumption that spillovers occur across regions with only a one quarter lag. We provide evidence, in the form of Moran's I statistic and Likelihood Ratio tests for the spatial lag coefficient λ that the spillovers are present within the quarter. These results imply that studies attempting to examine the monetary propagation process at the disaggregated state level should not ignore contemporaneous spillover across regions. In particular, estimated impulse response functions that omit spatial correlations are likely to underestimate the impact of monetary policy by shutting off an important channel for the real affects of monetary policy.

Including contemporaneous spillover acts as an additional channel with a multiplier effect for monetary policy. Thus, the estimated response to monetary policy shocks is larger, when the contemporaneous spillovers are allowed. The precise nature of this channel needs further investigation. Particularly interesting is why certain states respond stronger than others and which factors are important for explaining the spillovers. Future research will address these questions.

This paper has remained agnostic about the channels through which monetary policy propagates, and the nature of the spillovers, except that they have a spatial representation. Further studies of the transmission mechanism should attempt to more fully flesh out these channels. A particularly interesting avenue of research would consider the spillovers present in the residential housing markets and their relationship to monetary policy.

REFERENCES

[1] Fratantoni, Michael and Schuh, Scott, 2003. Monetary Policy, Housing, and Heterogeneous Regional Markets. *Journal of Money, Credit and Banking* 35 (4): 557-589

[2] Bernanke, Ben, S., and Alan S. Blinder, 1988. "Credit, Money and Aggregate Demand," *American Economic Review, papers and Proceedings* 78: 435-439

[3] Fotheringham, A. S. and M. E. O'Kelly, 1989. *Spatial Interaction Models: Formulations and Applications*. Dordrecht: Kluwer Academic.

[4] Kashyap, and Jeremy C. Stein, 1993. "Monetary Policy and Finance," American Economic Review 83: 78-98

[5] Kashyap, Anil K., Jeremy C. Stein, and D. Wilcox, 1995. "The Impact of Monetary Policy on Bank Balance Sheets," Carnegie-Rochester Conference Series on Public Policy 42: 151-195

[6] Carlino, Gerald and Defina, Robert, 1999. Do States Response Differently to Changes in Monetary Policy", *Business Review* (July/August) 17-27.

[7] Carlino, Gerald and Defina, Robert, 1998. The Differential Regional Effects of Monetary Policy. *The Review of Economics and Statistics* 80 (4): 572-587

[8] Christiano, Lawrence J., Martin Eichenbaum and Charles Evans, 1998. Monetary Policy Shocks: What Have We Learned and to What End? NBER Working Paper No. W6400.

[9] Owyang, M., and Wall, H., 2005. Structural Breaks and Regional Disparities in the Transmission of Monetary Policy. *Federal Reserve Bank of St. Louis Working Papers.*

[10] Lung-Fei Lee, 2007. GMM and 2SLS Estimation of Mixed Regressive, Spatial Autoregressive Models. *Journal of Econometrics* 137 (2): 489-514.

[11] Kelejian, H. H., and I. R. Prucha, 1998. A Generalized Spatial Two-Stage Least Squares Procedure for Estimating a Spatial Autoregressive Model with Autoregressive Disturbance. *Journal of Real Estate Finance and Economics* 17: 99-121.

7. APPENDIX 1: DERIVATION OF THE IMPULSE RESPONSE FUNCTION

In this section, we derive the relationship between the parameters of the reduced-form system and the counterparts in the structural system.

As in equation (8), the structural economic system is as

$$\begin{bmatrix} I_n - \Lambda W_n & 0_{48,1} & 0_{48,1} \\ -\lambda^p \widehat{W} & 1 & 0 \\ -\lambda^r \widehat{W} & -\delta & 1 \end{bmatrix} \begin{bmatrix} y_{nt} \\ p_t \\ r_t \end{bmatrix} = \begin{bmatrix} \Phi_1 + \Pi_1 W_n & \Gamma_1 & \Psi_1 \\ \pi_1^p \widehat{W} & \gamma_1^p & \psi_1^p \\ \pi_1^r \widehat{W} & \gamma_1^r & \psi_1^r \end{bmatrix} \begin{bmatrix} y_{nt-1} \\ p_{t-1} \\ r_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} \Phi_2 + \Pi_2 W_n & \Gamma_2 & \Psi_2 \\ \pi_2^p \widehat{W} & \gamma_2^p & \psi_2^p \\ \pi_2^r \widehat{W} & \gamma_2^r & \psi_2^r \end{bmatrix} \begin{bmatrix} y_{nt-2} \\ p_{t-2} \\ r_{t-2} \end{bmatrix}$$
$$+ \begin{bmatrix} \mu_n \\ \mu^p \\ \mu^r \end{bmatrix} I_{50} + \begin{bmatrix} u_{nt} \\ u_{pt} \\ u_{rt} \end{bmatrix},$$

and the relationship between the coefficient parameter matrices of our equation system and those of the general form of equation (1) is described as

$$A_0 = \begin{bmatrix} I_n - \Lambda W_n & 0 & 0 \\ -\lambda^p \widehat{W} & 1 & 0 \\ -\lambda^r \widehat{W} & -\delta & 1 \end{bmatrix}, A_1 = \begin{bmatrix} \Phi_1 + \Pi_1 W_n & \Gamma_1 & \Psi_1 \\ \pi_1^p \widehat{W} & \gamma_1^p & \psi_1^p \\ \pi_1^r \widehat{W} & \gamma_1^r & \psi_1^r \end{bmatrix},$$

$$A_2 = \begin{bmatrix} \Phi_2 + \Pi_2 W_n & \Gamma_2 & \Psi_2 \\ \pi_2^p \widehat{W} & \gamma_2^p & \psi_2^p \\ \pi_2^r \widehat{W} & \gamma_2^r & \psi_2^r \end{bmatrix}, \text{ and } M = \begin{bmatrix} \mu_n \\ \mu^p \\ \mu^r \end{bmatrix}.$$

In order to inverse A_0, we partition it into a block matrix as

$$A_0 = \left[\begin{array}{c|cc} (I_n - \Lambda W_n) & 0_{n\times 2} & \\ \hline -\lambda^p \widehat{W} & 1 & 0 \\ -\lambda^r \widehat{W} & -\delta & 1 \end{array}\right],$$

with $[(I_n - \Lambda W_n)]$ being an $n \times n$ matrix, $[0_{n\times 2}]$ being an $n \times 2$ zero matrix, $\begin{bmatrix} -\lambda^p \widehat{W} \\ -\lambda^r \widehat{W} \end{bmatrix}$ being a $2 \times n$ matrix, and $\begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}$ being a 2×2 matrix. Since the reduced-form equation system is

$$\begin{aligned} X_t &= b + B_1 X_{t-1} + B_2 X_{t-2} + V_t \\ &= A_0^{-1} M + A_0^{-1} A_1 X_{t-1} + A_0^{-1} A_2 X_{t-2} + A_0^{-1} U_t, \end{aligned}$$

where the following equalities hold for the relationship between the reduced-form and the structural form

$$\begin{aligned} b &= A_0^{-1} M, \\ B_1 &= A_0^{-1} A_1, \\ B_2 &= A_0^{-1} A_2 \\ V_t &= A_0^{-1} U_t. \end{aligned}$$

To establish a one to one mapping system between the elements in (b, B_1, B_2, V_t) and those in (M, A_0, A_1, A_2, U_t), we need to compute the inverse of A_0, we then use the following matrix inversion lemma for block form matrices .

If G is a block matrix as $G = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$,then the inverse of G is

$$G^{-1} = \begin{bmatrix} (A - BD^{-1}C)^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -D^{-1}C(A - BD^{-1}C)^{-1} & (D - CA^{-1}B)^{-1} \end{bmatrix}.$$

Given that B is a zero matrix, the inverse of G can be simplified as

$$G^{-1} = \begin{bmatrix} A^{-1} & 0 \\ -D^{-1}CA^{-1} & D^{-1} \end{bmatrix}.$$

To compute the inverse of A_0, let $A = (I_n - \Lambda W_n)$, $B = 0$, $C = \begin{bmatrix} -\lambda^p \widehat{W} \\ -\lambda^r \widehat{W} \end{bmatrix}$, and $D = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix}$. We have

$$
\begin{aligned}
A_0^{-1} &= \left[\begin{array}{c|c} (I_n - \Lambda W_n)^{-1} & 0 \\ \hline -\begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} -\lambda^p \widehat{W} \\ -\lambda^r \widehat{W} \end{bmatrix} (I_n - \Lambda W_n)^{-1} & \begin{matrix} 1 & 0 \\ \delta & 1 \end{matrix} \end{array}\right] \qquad (22) \\
&= \left[\begin{array}{c|c} (I_n - \Lambda W_n)^{-1} & 0 \\ \hline \begin{bmatrix} \lambda^p \widehat{W} \\ \lambda^p \delta \widehat{W} + \lambda^r \widehat{W} \end{bmatrix} (I_n - \Lambda W_n)^{-1} & \begin{matrix} 1 & 0 \\ \delta & 1 \end{matrix} \end{array}\right].
\end{aligned}
$$

To get the product of A_0^{-1} and A_1, we break down A_1 into a block matrix as

$$
A_1 = \left[\begin{array}{c|cc} \phi_1 I_n + \pi_1 W_n & \gamma_1 & \psi_1 \\ \hline \pi_1^p \widehat{W} & \gamma_1^p & \psi_1^p \\ \pi_1^r \widehat{W} & \gamma_1^r & \psi_1^r \end{array}\right],
$$

then

$$
A_0^{-1} A_1 = \left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right],
$$

where B_{11} denotes an $n \times n$ matrix as

$$
B_{11} = (I_n - \Lambda W_n)^{-1} (\phi_1 I_n + \pi_1 W_n),
$$

B_{12} denotes an $n \times 2$ matrix as

$$
B_{12} = (I_n - \Lambda W_n)^{-1} \begin{bmatrix} \gamma_1 & \psi_1 \end{bmatrix},
$$

B_{21} denotes a $2 \times n$ matrix as

$$
B_{21} = \begin{bmatrix} \lambda^p \widehat{W} \\ \lambda^p \delta \widehat{W} + \lambda^r \widehat{W} \end{bmatrix} (I_n - \Lambda W_n)^{-1} (\phi_1 I_n + \pi_1 W_n) + \begin{bmatrix} \pi_1^p \widehat{W} \\ \delta \pi_1^p \widehat{W} + \pi_1^r \widehat{W} \end{bmatrix},
$$

B_{22} denotes a 2×2 matrix as

$$
B_{22} = \begin{bmatrix} \lambda^p \widehat{W} \\ \lambda^p \delta \widehat{W} + \lambda^r \widehat{W} \end{bmatrix} (I_n - \Lambda W_n)^{-1} \begin{bmatrix} \gamma_1 & \psi_1 \end{bmatrix} + \begin{bmatrix} \gamma_1^p & \psi_1^p \\ \delta \gamma_1^p + \gamma_1^r & \delta \psi_1^p + \psi_1^r \end{bmatrix}.
$$

Similarly, to get the product of A_0^{-1} and A_2, we break A_2 into a block matrix as

$$
A_2 = \left[\begin{array}{c|cc} \phi_2 I_n + \pi_2 W_n & \gamma_2 & \psi_2 \\ \hline \pi_2^p \widehat{W} & \gamma_2^p & \psi_2^p \\ \pi_2^r \widehat{W} & \gamma_2^r & \psi_2^r \end{array}\right],
$$

and

$$
A_0^{-1} A_2 = \left[\begin{array}{c|c} C_{11} & C_{12} \\ \hline C_{21} & C_{22} \end{array}\right],
$$

where

$$
\begin{aligned}
C_{11} &= (I_n - \Lambda W_n)^{-1}(\phi_2 I_n + \pi_2 W_n), \\
C_{12} &= (I_n - \Lambda W_n)^{-1} \left[\begin{array}{cc} \gamma_2 & \psi_2 \end{array} \right], \\
C_{21} &= \left[\begin{array}{c} \lambda^p \widehat{W} \\ \lambda^p \delta \widehat{W} + \lambda^r \widehat{W} \end{array} \right] (I_n - \Lambda W_n)^{-1}(\phi_2 I_n + \pi_2 W_n) + \left[\begin{array}{c} \pi_2^p \widehat{W} \\ \delta \pi_2^p \widehat{W} + \pi_2^r \widehat{W} \end{array} \right], \\
C_{22} &= \left[\begin{array}{c} \lambda^p \widehat{W} \\ \lambda^p \delta \widehat{W} + \lambda^r \widehat{W} \end{array} \right] (I_n - \Lambda W_n)^{-1} \left[\begin{array}{cc} \gamma_2 & \psi_2 \end{array} \right] + \left[\begin{array}{cc} \gamma_2^p & \psi_2^p \\ \delta \gamma_2^p + \gamma_2^r & \delta \psi_2^p + \psi_2^r \end{array} \right].
\end{aligned}
$$

The product of A_0^{-1} and U_t is

$$
\begin{aligned}
A_0^{-1} U_t &= \left[\begin{array}{c|c} (I_n - \Lambda W_n)^{-1} & 0 \\ \hline \left[\begin{array}{c} \lambda^p \widehat{W} \\ \lambda^p \delta \widehat{W} + \lambda^r \widehat{W} \end{array} \right] (I_n - \Lambda W_n)^{-1} & \begin{array}{cc} 1 & 0 \\ \delta & 1 \end{array} \end{array} \right] \left[\begin{array}{c} u_{nt} \\ u_{pt} \\ u_{rt} \end{array} \right] \\
&= \left[\begin{array}{c} v_{nt} \\ v_{pt} \\ v_{rt} \end{array} \right],
\end{aligned}
$$

where

$$
\begin{aligned}
v_{nt} &= (I_n - \Lambda W_n)^{-1} u_{nt}, \\
v_{pt} &= \lambda^p \widehat{W} (I_n - \Lambda W_n)^{-1} u_{nt} + u_{pt}, \\
v_{rt} &= (\lambda^p \delta \widehat{W} + \lambda^r \widehat{W})(I_n - \Lambda W_n)^{-1} u_{nt} + \delta u_{pt} + u_{rt}.
\end{aligned}
$$

The impulse response function (IRF) captures the dynamic response of one variable to a shock to the structural system. This paper focuses on the state-level income response to the a structural policy shock. Mathematically, we have $\frac{\partial y_{it+s}}{\partial u_{rt}}$, where y_{it+s} denotes the real income growth rate in state i at time $t+s$, and u_{rt} denotes the structural policy shock at time t. Because $\frac{\partial v_{rt}}{\partial u_{rt}} = 1$,

$$
\frac{\partial y_{it+s}}{\partial u_{rt}} = \frac{\partial y_{it+s}}{\partial v_{rt}} \frac{\partial v_{rt}}{\partial u_{rt}} = \frac{\partial y_{it+s}}{\partial e_{rt}}.
$$

The above equation computes the response of income growth to a unit policy change. Thus, the response of income level to a unit policy shock is simply the summation of the growth changes over the past time as

$$
\frac{\partial Y_{it+s}}{\partial u_{rt}} = \sum_{k=t}^{k=t+s} \frac{\partial y_{ik}}{\partial u_{rt}},
$$

where Y_{it+s} denote the logarithm of the real income **level** in state i at time $t+s$, and y_{ij} denote the logarithm of the real income **growth** in state i at time k.