Spatial Dynamic Panel Model and System GMM: A Monte Carlo Investigation*

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Draft: May 2009

Abstract

This paper investigates the finite sample properties of estimators for spatial dynamic panel models in the presence of several endogenous variables. So far, none of the available estimators in spatial econometrics allows considering spatial dynamic models with one or more endogenous variables. We propose to apply system-GMM, since it can correct for the endogeneity of the dependent variable, the spatial lag as well as other potentially endogenous variables using internal and/or external instruments. The Monte-Carlo investigation compares the performance of spatial MLE, spatial dynamic MLE (Elhorst (2005)), spatial dynamic QMLE (Yu et al. (2008)), LSDV, difference-GMM (Arellano & Bond (1991)), as well as extended-GMM (Arellano & Bover (1995), Blundell & Bover (1998)) in terms of bias and root mean squared error. The results suggest that, in order to account for the endogeneity of several covariates, spatial dynamic panel models should be estimated using extended GMM. On a practical ground, this is also important, because system-GMM avoids the inversion of high dimension spatial weights matrices, which can be computationally demanding for large N and/or T.

Keywords: Spatial Econometrics, Dynamic Panel Model, System GMM, Maximum Likelihood, Monte Carlo Simulations

JEL classification: C15, C31, C33

*We thank Florian Pelgrin for helpful comments.

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1 Introduction

Although the econometric analysis of dynamic panel models has drawn a lot of attention in the last decade, econometric analysis of spatial and dynamic panel models is almost inexistent. So far, none of the available estimators allows to consider a dynamic spatial lag panel model with one or more endogenous variables (besides the time and spatial lag) as explanatory variables. Empirically, there are numerous examples where the presence of a dynamic process, spatial dependence and endogeneity might occur.

This is the case with the analysis of the determinants of complex Foreign Direct Investment (FDI). Complex FDI is undertaken by a multinational firm from home country i investing in production plants in several host countries to explore the comparative advantages of these locations (Baltagi et al. (2007)). This type of FDI can, thus, feature complementary/substitutive spatial dependence with respect to FDI to other host countries. The presence of complex FDI can be tested empirically by estimating a spatial lag model (as proposed by Blonigen et al. (2007)), which can also include a lagged dependent variable to account for the fact that FDI decisions are part of a dynamic process, i.e. more FDI in a host country seems to attract more FDI in this same host country. Additional endogenous variables like environmental stringency, tariffs or taxes can be included. These variables influence the decision of multinational to invest in a country but are also determined or influenced by the strategic interaction of these multinationals

The inclusion of the lagged dependent variable and additional endogenous variables in a spatial lag model invalidate the use of traditional spatial maximum likelihood estimators, that is why this model is estimated in several empirical studies (Kukenova & Monteiro (2009), Madariaga & Poncet (2007)) using the system generalized method of moments (GMM) estimator, developed by Arellano & Bover (1995) and Blundell & Bond (1998). The main argument of applying the extended GMM in a spatial context is that it corrects for the endogeneity of the spatial lagged dependent variable and other potentially endogenous explanatory variables. In addition, extended GMM is robust to some econometrics issues such as measurement error and weak instruments. It can also control for arbitrary heteroskedasticity and autocorrelation in the disturbance error. Its implementation is computationally tractable as it avoids the inversion of high dimension spatial weights matrix W and the computation of its eigenvalues.

Going beyond this intuitive motivation, we conduct an extensive Monte Carlo study comparing performance of extended GMM with performance of several spatial estimators (Spatial MLE (SMLE), Spatial Dynamic MLE (SDMLE) and Spatial Dynamic QMLE (SDQMLE)) in terms of bias and root mean squared error criteria. We verify the sensitivity of results to alternative distributional assumptions and alternative specification of the matrices of spatial weights. Finally, we check robustness of the estimators to misspecification of the matrices of spatial weights, which is an important issue in spatial econometrics.

The outline of the paper is as follows. The dynamic spatial lag model is defined in section 2. The Monte Carlo investigation is described and performed in section 3. Section 4 checks the robustness of the main results. Finally, section 5 concludes.

2 Spatial Dynamic Panel Model

Spatial data is characterized by the spatial arrangement of the observations. Following Tobler's First Law of Geography, *everything is related to everything else, but near things are more related than distant things*, the spatial linkages of the observations $i = 1, ..., N$ are measured by defining a spatial weight matrix[1], denoted by W_t for any year $t = 1, ..., T$:

$$W_t = \begin{pmatrix} 0 & w_t(d_{k,j}) & \cdots & w_t(d_{k,l}) \\ w_t(d_{j,k}) & 0 & \cdots & w_t(d_{j,l}) \\ \vdots & \vdots & \ddots & \vdots \\ w_t(d_{l,k}) & w_t(d_{l,j}) & \cdots & 0 \end{pmatrix}$$

where $w_t(d_{j,k})$ defines the functional form of the weights between any two pair of location j and k. In the construction of the weights themselves, the theoretical foundation for $w_t(d_{j,k})$ is quite general and the particular functional form of any single element in W_t is, therefore, not prescribed. In fact, the determination of the proper specification of W_t is one of the most difficult and controversial methodological issues in spatial data analysis. In empirical applications, geographical distance or economic distance are usually used.

[1] The modelling of spatial dependence can either be done with lattice models (i.e. weight matrix (Anselin (1988)) or geostatistical models (i.e. function of separative distances (Chen & Conley (2001)).

As is standard in spatial econometrics, for ease of interpretation , the weighting matrix W_t is row standardized so that each row in W_t sums to one. As distances are time-invariant, it will generally be the case that $W_t = W_{t+1}$. However, when dealing with unbalanced panel data, this is no longer true. Stacking the data first by time and then by cross-section, the full weighting matrix, W, is given by:

$$W = \begin{pmatrix} W_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & W_T \end{pmatrix}$$

The development of empirical spatial models is intimately linked to the recent progress in spatial econometrics. The basic spatial model was suggested by Cliff & Ord (1981), but it did not receive important theoretical extensions until the middle of the 1990s. Anselin (2001, 2006), Elhorst (2003b) & Lee & Yu (2009) provide thorough surveys of the different spatial models and suggest econometric strategies to estimate them.

2.1 Dynamic Spatial Lag Model

A general spatial dynamic panel model, also known as a spatial dynamic autoregressive model with spatial autoregressive error (SARAR(1,1)), can be described as follows:

$$\begin{aligned} Y_t &= \alpha Y_{t-1} + \rho W_{1t} Y_t + EX_t\beta + EN_t\gamma + \varepsilon_t \qquad (1) \\ \varepsilon_t &= \eta + \phi W_{2t}\varepsilon_t + v_t, \quad t = 1, ..., T \end{aligned}$$

where Y_t is a $N \times 1$ vector, W_{1t} and W_{2t} are $N \times N$ spatial weight matrices which are non-stochastic and exogenous to the model, η is the vector of country effect, EX_t is a $N \times p$ matrix of p exogenous explanatory variables ($p \geq 0$) and EN_t is a $N \times q$ matrix of q endogenous explanatory variables with respect to Y_t ($q \geq 0$). Finally, v_t is assumed to be distributed as $(\mathbf{0}, \Omega)$. By adding some restrictions to the parameters, two popular spatial model specifications can be derived from this general spatial model, namely the dynamic spatial lag model ($\phi = 0$) and the dynamic spatial error model ($\rho = 0$)[2].

[2] The analysis of the spatial error panel model and the spatial lag with spatial error model is beyond the scope of this paper. For further details, see Elhorst (2005), Kapoor et al. (2007), Kelejian & Prucha (2007), Mutl & Pfaffermeier (2008) as well as Lee & Ju (2009).

The spatial lag model accounts directly for relationships between dependent variables that are believed to be related in some spatial way. Somewhat analogous to a lagged dependent variable in time series analysis, the estimated "spatial lag" coefficient[3] characterizes the contemporaneous correlation between one cross-section and other geographically-proximate cross-sections. The following equation gives the basic spatial dynamic panel specification, also known as the "time-space simultaneous" model (Anselin (1988, 2001))[4]:

$$Y_t = \alpha Y_{t-1} + \rho W_t Y_t + EX_t \beta + EN_t \gamma + \eta + v_t \tag{2}$$

The spatial autoregressive coefficient (ρ) associated with $W_t Y_t$ represents the effect of the weighted average ($w_t(d_{ij})$ being the weights) of the neighborhood, i.e. $[W_t Y_t]_i = \sum_{j=1..N_t} w_t(d_{ij}) \cdot Y_{jt}$. The spatial lag term allows to determine if the dependent variable Y_t is (positively/negatively) affected by the Y_t from other close locations weighted by a given criterion (usually distance or contiguity). In other words, the spatial lag coefficient captures the impact of Y_t from neighborhood locations. Let $\omega_{\min}$ and $\omega_{\max}$ be the smallest and highest characteristic root of the spatial matrix W, then this spatial effect is assumed to lie between $\frac{1}{\omega_{\min}}$ and $\frac{1}{\omega_{\max}}$. Most of the spatial econometrics literature constrains the spatial lag to lie between -1 and +1. However, this might be restrictive, because if the spatial matrix is row-normalized, then the highest characteristic root is equal to unity ($\omega_{\max} = 1$), but the smallest eigenvalue can be bigger than -1, which would lead the lower bound to be smaller than -1.

Given that expression (2) is a combination of a time and spatial autoregressive models, we need to ensure that the resulting process is stationary. The stationarity restrictions in this model are stronger than the individual restrictions imposed on the coefficients of a spatial or dynamic model. The process is covariance stationary if $\left|(I_N - \rho W_t)^{-1} \alpha\right| < 1$, or, equivalently, if

$$\begin{aligned} |\alpha| &< 1 - \rho\omega_{\max} \quad \text{if } \rho \geq 0 \\ |\alpha| &< 1 - \rho\omega_{\min} \quad \text{if } \rho < 0 \end{aligned}$$

[3]The spatial autoregressive term is also referred as endogenous interation effects in social economics or as interdependence process in political science.

[4]Beside the "time-space simulatenous" model, Anselin distinguishes three other distinct spatial lag panel models: the "pure space recursive" model which only includes a lagged spatial lag coefficient; the "time-space recursive" specification which considers a lagged dependent variable as well as a lagged spatial lag (see Korniotis (2007)); and the "time-space dynamic" model, which includes a time lag, a spatial lag and a lagged spatial lag.

From an econometric viewpoint, equation (2) faces simultaneity and endogeneity problems, which in turn means that OLS estimation will be biased and inconsistent (Anselin (1988)). To see this point more formally, note that the reduced form of equation(2) takes the following form:

$$Y_t = (I_N - \rho W_t)^{-1} (\alpha Y_{t-1} + EX_t\beta + EN_t\gamma + \eta + v_t)$$

Each element of Y_t is a linear combination of all of the error terms. Moreover, as pointed out by Anselin (2003), assuming $|\rho| < 1$ and each element of W_t is smaller than one imply that $(I_N - \rho W_t)^{-1}$ can be reformulated as a Leontief expansion $(I_N - \rho W_t)^{-1} = I + \rho W_t + \rho^2 W_t^2 + \ldots$. Accordingly, the spatial lag model features two types of global spillovers effects: a multiplier effect for the predictor variables as well as a diffusion effect for the error process. Since the spatial lag term $W_t Y_t$ is correlated with the disturbances, even if v_t are independently and identically distributed, it must be treated as an endogenous variable and proper estimation method must account for this endogeneity.

Despite the fact that dynamic panel models have been the object of recent important developments (see the survey by Baltagi & Kao (2000) or Phillips & Moon (2000)), econometric analysis of spatial dynamic panel models is almost inexistent. In fact, there is only a limited number of available estimators that deal with spatial and time dependence in a panel setting. Table 1 summarizes the different estimators proposed in the literature.

In the absence of spatial dependence, there are three main types of estimators available to estimate a dynamic panel model. The first type of estimators consists of estimating an unconditional likelihood function (Hsiao et al. (2002)). The second type of procedure corrects the bias associated with the least square dummy variables (LSDV) estimator (Bun & Carree (2005)). The last type, which is the most popular, relies on GMM estimators, like difference GMM (Arellano & Bond (1992)), system GMM (Arellano & Bover (1995), Blundell & Bond (1998)) or continuously updated GMM (Hansen et al. (1996)).

Assuming all explanatory variables are exogenous beside the spatial autoregressive term, the spatial lag panel model without any time dynamic is usually estimated using spatial maximum likelihood (Elhorst (2003b)) or spatial two-stage least squares methods (S2SLS) (Anselin (1988) (2001)). The ML approach consists of estimating the spatial coefficient by maximizing the non-linear reduced form of the spatial lag model.

Table 1: Spatial Dynamic Estimators Survey

Model	Estimation Methods	Endogenous
$Y_t = \alpha Y_{t-1} + \beta EX_t + \epsilon_t$	GMM (Arellano et al. (1991) (1995), Blundell et al. (1998), Hansen et al. (1996)) MLE/Minimum Distance (Hsiao, Pesaran & Tahmiscioglu (2002)) CLSDV (Kiviet (1995), Hahn & Kuersteiner (2002) Bun & Carree (2005))	Y_{t-1};
$Y_t = \alpha Y_{t-1} + \beta EX_t + \gamma EN_t + \epsilon_t$	GMM (Arellano et al. (1991) (1995), Blundell et al. (1998), Hansen et al. (1996))	$Y_{t-1}; EN_t$
$Y_t = \alpha Y_{t-1} + \rho WY_{t-1} + \beta EX_t + \gamma EN_t + \epsilon_t$	LSDV-IV (Korniotis (2008))	$Y_{t-1}; EN_t$
$Y_t = \rho WY_t + \beta EX_t + \epsilon_t$	Spatial-MLE / Spatial 2SLS(Anselin (1988) (2001), Elhorst (2003)) Minimum Distance (Azomahou (2008))	WY_t
$Y_t = \rho WY_t + \beta EX_t + \gamma EN_t + \epsilon_t$	Spatial 2SLS (Dall'erba & Le Gallo (2007))	$WY_t; EN_t$
$Y_t = \alpha Y_{t-1} + \rho WY_t + \beta EX_t + \epsilon_t$	Spatial Dynamic MLE (Elhorst (2003b, 2005, 2008)) Spatial Dynamic QMLE (Yu, de Jong & Lee (2007) (2008), Lee & Yu (2007)) C2SLSDV (Beenstock & Felsenstein (2007)) Spatial MLE-GMM / Spatial MLE-Spatial Dynamic MLE (Elhorst (2008))	$WY_t; Y_{t-1}$
$Y_t = \alpha Y_{t-1} + \rho WY_t + \beta EX_t + \gamma EN_t + \epsilon_t$	System-GMM (Arellano & Bover (1995), Blundell & Bond (1998))	$WY_t; Y_{t-1}; EN_t$

The spatial 2SLS uses the exogenous variables and their spatially weighted averages (EX_t, $W_t \cdot EX_t$) as instruments[5]. When the number of cross-sections is larger than the period sample, Anselin (1988) suggests to estimate the model using MLE, 2SLS or 3SLS in a spatial seemingly unrelated regression (SUR) framework. More recently, Azomahou (2008) proposes to estimate a spatial panel autoregressive model applying a two-step minimum distance estimator. In the presence of endogenous variables, Dall'erba & Le Gallo (2007) suggest to estimate a spatial lag panel model, which includes several endogenous variables but no lagged dependent variable, by applying spatial 2SLS with lower orders of the spatially weighted sum of the exogenous variables as instrument for the spatial autoregressive term[6].

In a dynamic context, the estimation of spatial lag panel models is usually based on a ML function. Elhorst (2003a, 2005) proposes to estimate the unconditional loglikelihood function of the reduced form of the model in first-difference. While the absence of explanatory variables besides the time and spatial lags leads to an exact likelihood function, this is no longer the case when additional regressors are included. Moreover, when the sample size T is relatively small the initial observations contribute greatly to the overall likelihood. That is why the pre-sample values of the explanatory variables and likelihood function are approximated using the Bhargava & Sargan approximation or the Nerlove & Balestra approximation. More recently, Yu et al. (2008) provide a theoretical analysis on the asymptotic properties of the quasi-maximum likelihood (Spatial Dynamic QML), which relies on the maximization of the concentrated likelihood function of the demeaned model. They show that the limit distribution is not centered around zero and propose a bias-corrected estimator[7]. Beside the fact that Yu et al. (2008) do not assume normality, the main difference with Elhorst's ML estimators lies in the asymptotic structure. Elhorst considers fixed T and large N $(N \to \infty)$, while Yu et al. assume large N and T $(N \to \infty;\ T \to \infty)$. Consequently, the way the individual effects are taken out differs: Elhorst considers first-difference variables, while Yu et al.

[5]In a cross-section setting, Kelejian & Prucha (1998) suggest also additional instruments ($W_t^2 EX_t$, $W_t^3 EX_t$, ...). Lee (2003) shows that the estimator proposed by Kelejian & Prucha is not an asymptotically optimal estimator and suggests a three-steps procedure with an alternative instrument for the spatial autoregressive coefficient in the last step ($W_t \cdot \left(I_N - \widetilde{\widetilde{\rho}} W_t\right)^{-1} \cdot EX_t \widetilde{\widetilde{\beta}}$, where $\widetilde{\widetilde{\rho}}$ and $\widetilde{\widetilde{\beta}}$ are estimates obtained using the S2SLS proposed by Kelejian & Prucha (1998)).

[6]Recently, Fingleton & Le Gallo (2008) propose an extended feasible generalized spatial two-stage least squares estimator for spatial lag models with several endogenous variables and spatial error term in a cross-section framework.

[7]In two other related working papers, Lee & Yu (2007) and Yu et al.(2007) investigate the presence of non-stationarity and time fixed effects, respectively, in a spatial dynamicpanel framework.

demean the variables. Assuming large T avoids the problem associated with initial values and the use of approximation procedures. Finally, Yu et al's approach allows to recover the estimated individual effects, which is not the case with the estimator proposed by Elhorst.
In a recent paper, Elhorst (2008) analyzes the finite sample performance of several estimators for a spatial dynamic panel model with only exogenous variables. The estimators considered are the Spatial MLE, Spatial Dynamic MLE and GMM. His Monte Carlo study shows that Spatial Dynamic MLE has the better overall performance in terms of bias reduction and root mean squared errors (RMSE), although the Spatial MLE presents the smallest bias for the spatial autoregressive coefficient. Based on these results, Elhorst proposes two mixed estimators, where the spatial lag dependent variable is based on the spatial ML estimator and the remaining parameters are estimated using either GMM or Spatial Dynamic ML conditional on the spatial ML's estimate of the spatial autoregressive coefficient. These two mixed estimators outperform the original estimators. The mixed Spatial MLE/Spatial Dynamic MLE estimator shows superior performance in terms of bias reduction and RMSE in comparison with mixed Spatial MLE/GMM. However, the latter can be justified on a practical ground if the number of cross-sections in the panel is large, since the time needed to compute Spatial MLE/Spatial Dynamic MLE is substantial. In a spatial vector autoregression (VAR) setting, Beenstock & Felsenstein (2007) suggest a two-step procedure. The first step consists of applying LSDV to the model without the spatial lag and computing the fitted values ($\widehat{Y_t}$). Then, in the second step, the full model is also estimated using LSDV, but with $W_t\widehat{Y_t}$ as instrument for W_tY_t. Finally, the authors suggest to correct the bias of the lagged dependent variable by using the asymptotic bias correction defined by Hsiao(1986).

If one is willing to consider some explanatory variables as potentially endogenous in a dynamic spatial panel setting, then no spatial estimator is currently available. In many empirical applications, endogeneity can arise from measurement errors, variables omission or the presence of simultaneous relationship(s) between the dependent and the covariate(s). The main drawback of applying SMLE, SDMLE or SDQMLE is that, while the spatial autoregressive coefficient is considered endogenous, no instrumental treatment is applied to other potential endogenous variables. This can lead to biased estimates, which would invalidate empirical results. This is confirmed by the Monte Carlo analysis performed in the next section.

2.2 System GMM

Empirical papers dealing with a spatial dynamic panel model with several endogenous variables usually apply system-GMM (see for example Madriaga & Poncet (2007), Foucault, Madies & Paty (2008), or Hong, Sun & Li (2008)). Back in 1978, Haining proposed to instrument a first order spatial autoregressive model using lagged dependent variables. While this method is not efficient in a cross-section setting, because it does not use efficiently all the available information (Anselin (1988)), this is no longer necessarily the case in a panel framework. The bias-corrected LSDV-IV estimator proposed by Korniotis (2007) is in line with this approach and considers lagged spatial lag and dependent variable as instruments. Accordingly, the use of system GMM might be justified in this trade-off situation, since the spatial lag would be instrumented by lagged values of the dependent variable, spatial autoregressive variable as well as exogenous variables[8].

For simplicity, equation (2) is reformulated for a given cross-section i $(i = 1, .., N)$ at time t $(t = 1, .., T)$:

$$Y_{it} = \alpha Y_{it-1} + \rho \left[W_t Y_t\right]_i + EX_{it}\beta + EN_{it}\gamma + \eta_i + v_{it} \tag{3}$$

According to the GMM procedure, one has to eliminate the individual effects (η_i) correlated with the covariates and the lagged dependent variable, by rewriting equation(3) in first difference for individual i at time t:

$$\triangle Y_{it} = \alpha \triangle Y_{it-1} + \rho \triangle \left[W_t Y_t\right]_i + \triangle EX_{it}\beta + \triangle EN_{it}\gamma + \triangle v_{it} \tag{4}$$

Even if the fixed effects (within) estimator cancels the individual fixed effcts(η_i), the lagged endogenous variable $(\triangle Y_{it-1})$ is still correlated with the idiosyncratic error terms (v_{it}). Nickell (1981) as well as Anderson & Hsiao (1981) show that the within estimator has a bias measured by O$(\frac{1}{T})$ and is only consistent for large T. Given that this condition is usually not satisfied, the GMM estimator is also biased and inconsistent. Arellano & Bond (1991) propose the following moment conditions associated with equation (4):

$$E\left(Y_{i,t-\tau} \triangle v_{it}\right) = 0; \quad \text{for } t = 3, ..., T \text{ and } 2 \leq \tau \leq t-1 \tag{5}$$

[8] Badinger et al. (2004) recommend to apply system GMM, once the data has been spatially filtered. This approach can be consider only when spatial depence is viewed as a nuisance parameter.

But the estimation based only on these moment conditions (5) is insufficient, if the strict exogeneity assumption of the covariates (EX_{it}) has not been verified. The explicative variables constitute valid instruments to improve the estimator's efficiency, only when the strict exogeneity assumption is satisfied:

$$E\left(EX_{i\tau}\triangle v_{it}\right)=0;\quad \text{for } t=3,...,T \;\text{ and }\; 1\leq\tau\leq T \tag{6}$$

However, the GMM estimator based on the moment conditions (5) and (6) can still be inconsistent when $\tau < 2$ and in presence of inverse causality, i.e. E$(EX_{it}v_{it}) \neq 0$. In order to overcome this problem, one can assume that the covariates are weakly exogenous for $\tau < t$, which means that the moment condition (6) can be rewritten as:

$$E\left(EX_{i,t-\tau}\triangle v_{it}\right)=0;\quad \text{for } t=3,...,T \;\text{ and }\; 1\leq\tau\leq t-1 \tag{7}$$

For the different endogenous variables, including the spatial lag, the valid moment conditions are

$$E\left(EN_{i,t-\tau}\triangle v_{it}\right) = 0;\quad \text{for } t=3...T \;\text{ and }\; 2\leq\tau\leq t-1 \tag{8}$$

$$E\left(\left[W_{t-\tau}Y_{t-\tau}\right]_i\triangle v_{it}\right) = 0;\quad \text{for } t=3...T \;\text{ and }\; 2\leq\tau\leq t-1 \tag{9}$$

For small samples, this estimator can still yield biased coefficients. Blundell & Bond (1998) show that the imprecision of this estimator is bigger as the individual effects are important and as the variables are persistent over time. Lagged levels of the variables are weak instruments for the first differences in such case**.** To overcome this limits, the authors propose the system GMM, which estimate simultaneously equation(3)and equation (4). The extra moment conditions for the extended GMM are thus:

$$E\left(\triangle Y_{i,t-1}v_{it}\right) = 0;\quad \text{for } t=3,...,T \tag{10}$$

$$E\left(\triangle EX_{it}v_{it}\right) = 0;\quad \text{for } t=2,...,T \tag{11}$$

$$E\left(\triangle EN_{it-1}v_{it}\right) = 0;\quad \text{for } t=3,...,T \tag{12}$$

$$E\left(\triangle\left[W_{t-1}Y_{t-1}\right]_i v_{it}\right) = 0;\quad \text{for } t=3,...,T \tag{13}$$

The consistency of the SYS-GMM estimator relies on the validity of these moment conditions, which depends on the assumption of absence of serially correlation of the level residuals and the exogeneity of the explanatory variables. Therefore, it is necessary to apply specification tests to ensure that these assumptions are justified. More generally, one should keep in mind that the estimation of the spatial autoregressive coefficient via SYS-GMM, although "potentially" consistent, is usually not the most efficient one. Efficiency relies on the "proper" choice of instruments, which is not an easy task to determine. Arellano & Bond suggest two specification tests in order to verify the consistency of the GMM estimator. First, the overall validity of the moment conditions is checked by the Sargan/Hansen test. Second, the Arellano-Bond test examines the serial correlation property of the level residuals.

Another issue lies in the fact that the instrument count grows as the sample size T rises. A large number of instruments can overfit endogenous variables (i.e. fail to correct for endogeneity) and leads to inaccurate estimation of the optimal weight matrix, downward biased two-step standard errors and wrong inference in the Hansen test of instruments validity. Okui (2009) demonstrates that the bias of extended GMM does not result from the total number of instruments, but from the number of instruments for each equation. As pointed out by Roodman (2009), it is advisable to restrict the number of instruments by defining a maximum number of lags and/or by collapsing the instruments[9]. The *collapse* option consists of combining instruments through addition into subsets. For instance, if the instruments are collapsed, the moment condition (5) becomes:

$$E\left(Y_{i,t-\tau}\triangle v_{it}\right)=0; \quad \text{for } 2\leq\tau\leq t-1 \tag{14}$$

This modified moment condition still imposes the orthogonality of $Y_{i,t-\tau}$ and $\triangle v_{it}$, but rather to hold for each t and τ, it is only valid for each τ. Roodman (2009) shows that collapsed instruments lead to less biased estimates, although the associated standard errorstend to increase. GMM results for the remaining of the paper are, thus, based on collapsed instruments[10].

3 A Monte-Carlo Study

In this section, we investigate the finite sample properties of several estimators including Spatial MLE, Spatial Dynamic MLE and Spatial Dynamic QMLE, LSDV, difference GMM and extended GMM to account for the endogeneity of the spatial lag as well as an additional regressor in a dynamic panel data context using Monte-Carlo simulations[11].

[9] This approach has been adopted in several empirical papers, including Beck & Levine (2004).
[10] See appendix 6.A for further details on extended GMM and appendix 6.B for spatial ML estimators.
[11] All simulations are performed using Matlab R2008b.

Simulation studies already showed that the bias associated with the spatial lag is rather small (Franzese & Hays (2007), Elhorst (2008)), but none analyzes the consequences of an additional endogenous explanatory variable in a spatial dynamic context. The data generating process (DGP) is defined as follows:

$$\begin{aligned} Y_{it} &= \alpha Y_{i,t-1} + \rho \left[W Y_t \right]_i + \beta E X_{it} + \gamma E N_{it} + \eta_i + v_{it} & (15) \\ E X_{it} &= \delta E X_{i,t-1} + u_{it} & (16) \\ E N_{it} &= \lambda E N_{i,t-1} + \psi \eta_i + \theta v_{it} + e_{it} & (17) \end{aligned}$$

with $\eta_i \sim N\left(0, \sigma_\eta^2\right)$; $v_{it} \sim N\left(0, \sigma_v^2\right)$; $u_{it} \sim N\left(0, \sigma_u^2\right)$; $e_{it} \sim N\left(0, \sigma_e^2\right)$.

In order to avoid results being influenced by initial observations, the covariates Y_{i0}, EX_{i0} and EN_{i0} are set to 0 for all i and each variable is generated $(100 + T)$ times according to their respective DGP. The first 100 observations are then discarded. Note that the dependent variable is generated according to the reduced form of equation (14):

$$Y_{it} = \left(1 - \rho \left[W\right]_i\right)^{-1} \left[\alpha Y_{i,t-1} + \beta E X_{it} + \gamma E N_{it} + \eta_i + v_{it}\right] \quad (18)$$

Following Kapoor et al. (2007) and Kelejian & Prucha (1999), we consider four different types of spatial weight matrix. In each case, the matrix is row-standardized so that all non zero elements in each row sum to one. The matrices considered rely on a perfect "idealized" circular world. More precisely, the three "theoretical" spatial matrices, referred as "1 ahead and 1 behind", "3 ahead and 3 behind" and "5 ahead and 5 behind", respectively, are characterized by different degree of sparseness. Each are such that each location is related to the one/three/five locations immediately before and after it, so that each nonzero elements are equal to $0.5/0.\overline{3}/0.1$, respectively. In addition, as a robustness check, we consider real data on the distance between capitals among 224 countries[12]. In order to avoid giving some positive weight to very remote countries (with weaker cultural, political and economic ties), we consider the negative exponential weighting scheme. This is done by dividing the distance between locations j and k by the minimum distance within the region r (where location j lies within region r): $w\left(d_{j,k}\right) = \exp\left(-d_{j,k}/MIN_{r,j}\right)$ if $j \neq k$.

[12] The data is taken from CEPII database: http://www.cepii.fr/anglaisgraph/bdd/distances.htm.

The Monte Carlo experiments rely on the following designs:

$$T \in \{10, 20, 30, 40\}\,; \quad N \in \{20;\ 30;\ 50;\ 70\}\,;$$
$$\alpha \in \{0.2;\ 0.4;\ 0.5;\ 0.7\}\,; \quad \rho \in \{0.1;\ 0.3;\ 0.5;\ 0.7\}\,;$$
$$\beta = 1;\ \delta = 0.65;\ \gamma = 0.5;\ \lambda = 0.45;\ \psi = 0.25;\ \theta = 0.6;$$
$$\sigma_u^2 = 0.05\ \sigma_v^2 = 0.05;\ \sigma_e^2 = 0.05;$$

In order to ensure stationarity, only designs which respect the restrictions $|\alpha| < 1 - \rho\omega_{\max}$ if $\rho \geq 0$ or $|\alpha| < 1 - \rho\omega_{\min}$ if $\rho < 0$ are considered. The total number of designs is restricted to 160. For each of these designs, we performed 1000 trials. Note that for each design, the initial conditions and spatial weight matrices are generated once.

As a measure of consistency, we consider the root mean square error (RMSE). Theoretically, RMSE is defined as the square root of the sum of the variance and the squared bias of the estimator. Therefore, the RMSE assesses the quality of an estimator in terms of its variation and unbiasedness. We also consider the approximated RMSE given in Kelejian & Prucha (1999) and Kapoor et al. (2007), which converges to the standard RMSE under a normal distribution:

$$RMSE = \sqrt{bias^2 + \left(\frac{IQ^2}{1.35}\right)^2}$$

where the *bias* is the difference between the true value of the coefficient and the median of the estimated coefficients, and IQ is the difference between the 75% and 25% quantile. This definition has the advantage of being more robust to outliers that may be generated by the Monte-Carlo simulations.

The Monte Carlo investigation highlights several important facts[13]. First, the results are qualitatively similar with respect to different spatial weight schemes, that is why for sake of brevity we only present the results for the "1 ahead and 1 behind" weighting matrix. Second, the results in terms of bias and efficiency depend on the values assigned to the spatial and time lag parameters. Since the results based on RMSE and approximated RMSE are qualitatively similar, we only present the results associated with standard RMSE. In order to assess the global properties of the estimators, we first discuss the results obtained by averaging over the parameters values of the time and spatial lag coefficients..

[13] The full results are given in appendix 6.C and 6.D.

3.1 Extended GMM vs. Difference GMM

We first investigate the consistency and efficiency of the difference and system GMM estimators in a spatial dynamic panel framework. We consider two-step GMM based on a collapsed instruments structure. To avoid imprecise estimate of the optimal weight matrix (when $T > N$), we restrict the lags to two and three. In other words, each endogenous variables (i.e. Y_{t-1}, WY_t, EN_t) is instrumented with their 2nd and 3rd lags values (using the *collapse* option) and the exogenous variables X_t.

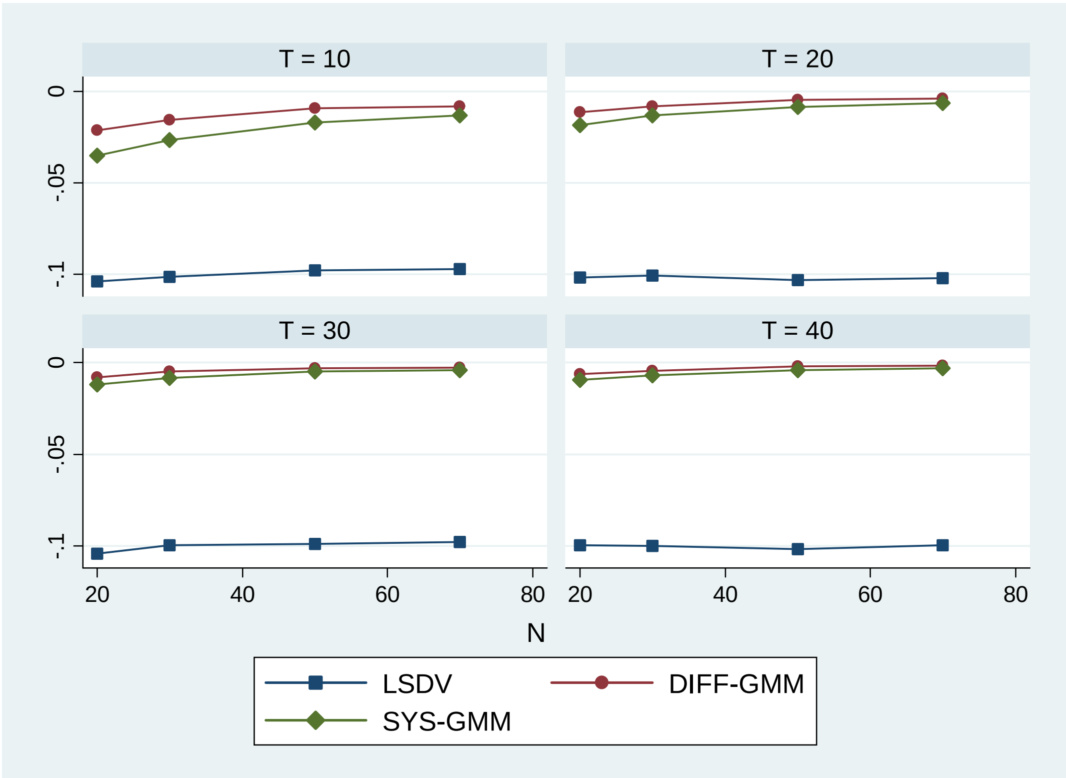

Figure 1: Bias LSDV *vs.* GMM

We consider global consistency and efficiency of GMM estimators: the bias and RMSE results are averaged over the whole range of parameters. For illustrative purpose, we compare the performance of GMM estimators with respect to LSDV estimator. As figure 1 and 2 indicate, system and difference GMM outperform the fixed effect estimator in terms of bias and efficiency. In fact, LSDV estimator has a negative bias which does not decrease with the size of the panel, invalidating LSDV as a consistent and efficient estimator. Consequently, spatial dynamic panel model should definitively not be estimated with LSDV. The differences in terms of performance between difference and system GMM are relatively marginal. Hayakawa(2007) shows that difference and level GMM estimators can be more biased than system GMM, because the bias of extended-GMM is the sum of two elements. The first one is the weighted sum of the bias associated with difference and level GMM, while the second one originates from using the level and difference estimators jointly. Since the bias of difference and level GMM evolve in opposite direction, the first component of the bias of system GMM will tend to be small due to a partially cancelling out effect. In addition, the weight of the first element plays also a role in the difference of the magnitudes of the biases. The latter could explain why both GMM estimators performs almost equivalently. However as depicted in figure 2, system GMM seems to be more efficient in small sample. That is why, we decide to focus on extended GMM.

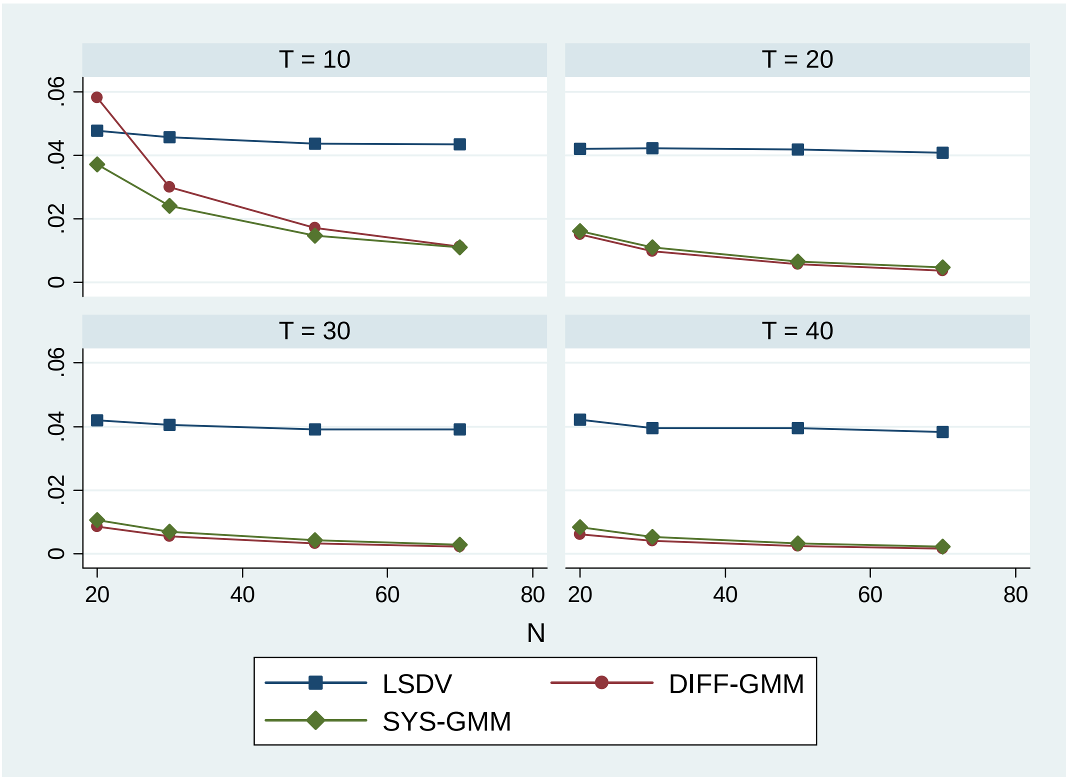

Figure 2: RMSE LSDV *vs.* GMM

3.2 Extended GMM vs. Spatial Estimators

In this section, we compare the global performance of extended GMM with respect to spatial ML estimators[14]. We consider the average value of the bias and RMSE for the different values of the autoregressive and spatial autoregressive coefficient. For each parameter, figure 3 plots the averaged bias of the respective estimators over different range of cross-sectional dimension, N[15]. The upper panels of figure 3 display the average bias associated with the autoregressive and spatial autoregressive variables respectively, while the lower panels present the average bias for the endogenous and exogenous variables. Although the bias associated with the spatial estimators seems to be independent of the cross-section dimension, it does decrease as the time dimension increases. This corroborates the finding in Yu et al., that the spatial estimators remain slightly biased as the number of cross-section increases (see table 2 and 3 in Yu et al. (2008)). Unlike spatial maximum likelihood estimators, extended GMM's bias decreases as N and/or T increase.

As displayed by the plot on the top left of figure 3, all estimators tend to underestimate the autoregressive parameter, (e.g. positive bias), although extended GMM exhibits the smallest bias. The opposite happens for the spatial autoregressive parameter. Note that the spatial estimators tend to dominate the extended GMM in relatively small sample. This is not surprising since spatial estimators explicitly account for the spatial structure of the data by estimating the reduced form of the model. Still as the time dimension increases, extended GMM tends to outperform the spatial estimators. The exogenous variable is also overestimated by all the spatial estimators, although its estimation with system GMM clearly dominates the other estimators. Most importantly, the plot on the bottom left of figure 3 shows how important it is to correct for the endogeneity. In fact, if not corrected, the bias associated with the endogenous variable can represent more than 60% of the true value of the parameter, which is unacceptable. Moreover, the magnitude of the bias of the endogenous covariate does not seem to depend on the sample dimension (N and T). This finding suggests that estimating a spatial dynamic panel model with endogenous variables using traditional spatial estimators would not be advisable. Overall, extended GMM exhibits superior performance in terms of bias. Beside SYS-GMM, spatial dynamic QMLE displays the lower bias for all coefficients.

[14] Note that the spatial estimators rely on a numerical optimization which can lead to non-convergent solution. When convergence is not obtained, the trial is dropped from the analysis. See appendix 6.B for further details about the implementation of the spatial estimators.

[15] We take average value of the bias across different designs of autoregressive and spatial autoregresive parameters

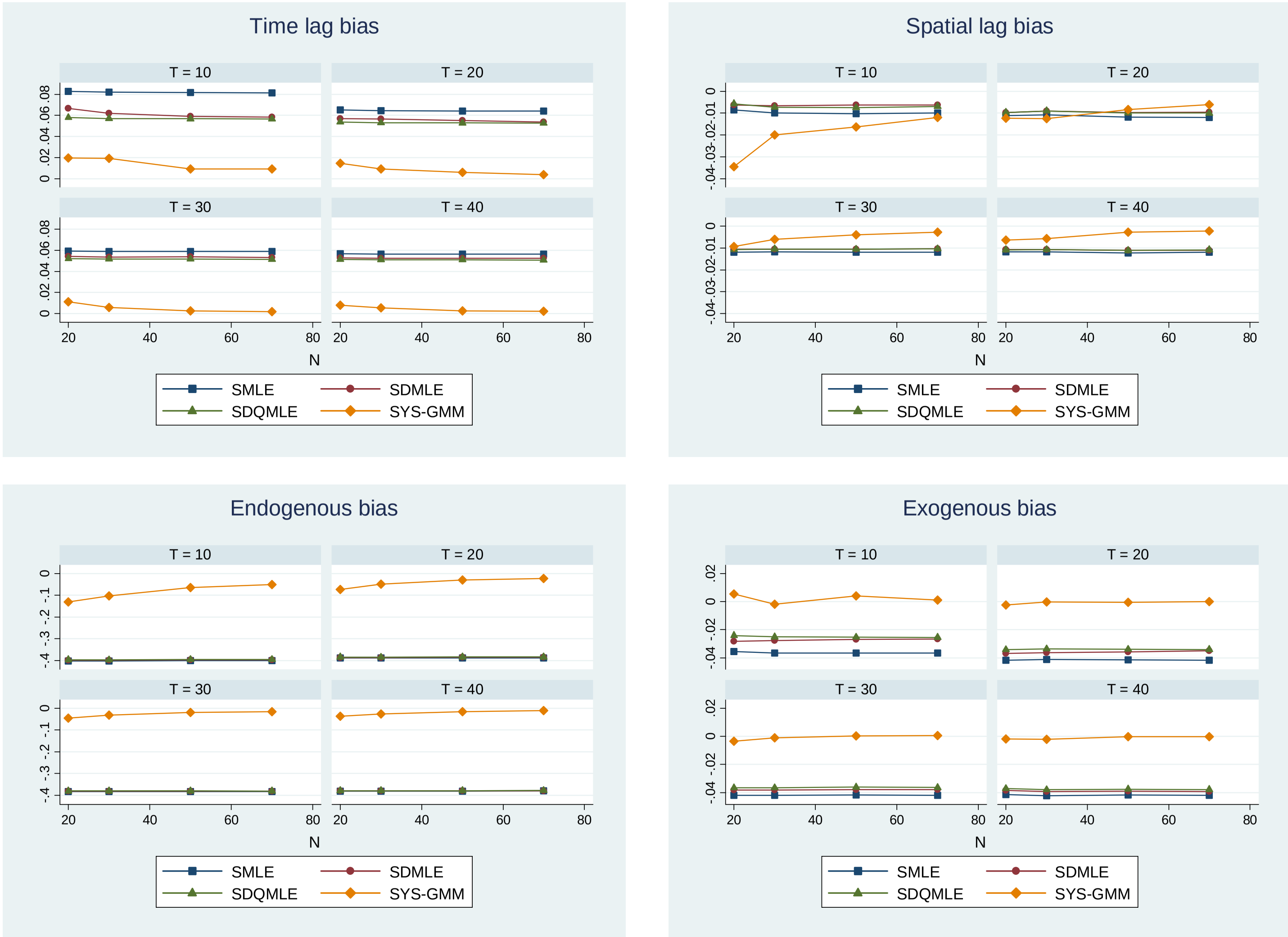

Figure 3: Bias Extended GMM *vs.* Spatial Estimators

Figure 4 depicts the performance of the estimators in terms of efficiency according to RMSE. The results are slightly different from the ones we obtained in the analysis of the bias. Despite the fact that spatial ML estimators yield more bias, they tend to be more efficient than extended GMM in small samples for most parameters, except the endogenous parameter. This is confirmed by figure 5, which plots the histogram distribution for each estimated parameter based on 1000 trials for $N = 70$ and $T = 20$. However, as the panel size increases, the performance of GMM converges to the same level of efficiency of the spatial ML estimators. Interestingly, the estimation in moderate samples of the time lag and exogenous variables with SYS-GMM can be even more efficient than with spatial estimators. In other words, extended GMM might be more consistent and efficient than spatial ML estimators. This result might seems to be a paradox with respect to the accepted notion that ML is more efficient than GMM, but it is not. Actually, this finding is an extension to the panel framework of Das et al.'s (2003) conclusion that FG2SLS can outperform ML estimators in moderate cross-section samples. The main reason for this seemingly contradictory result is that the spatial ML approach requires the estimation of more parameters than does extended GMM. ML involves the estimation of the additional parameter σ^2. Moreover, Elhorst's approach implies the estimation of the initial condition restrictions, while Yu et al's method is explicitly designed for large N and T. Therefore, the classical arguments relating to relative efficiency do not apply here. In addition, as figures 3 and 4 indicate, the spatial estimators seem to reach efficiency in relatively small samples (making the slope of RMSE and bias almost flat), while extended GMM tends to gain efficiency in moderate sample (the slope of RMSE and bias being more steeper).

As we already mentioned it in the bias analysis, the estimation of the endogenous covariate is clearly more efficient with extended GMM than with any of the spatial maximum likelihood estimators. The slope of the RMSE line in figure 4 is almost null for the QMLE and MLE, which suggests that increasing the dimension sample cannot improve efficiency of the estimate of the endogenous variable. In other words, the use of spatial ML estimators is not recommended in the presence of endogenous or predetermined variable. Among the spatial estimators, SDQMLE is again the best one in terms of efficiency. Note, that the estimation of the spatial lag parameter by simple spatial MLE is as efficient as by the other spatial estimators. This observation was highlighted by Elhorst (2008) who suggests to estimate a dynamic panel model with exogenous variables by first estimating the spatial lag with SMLE and then estimating the remaining parameters using either SDMLE or GMM.

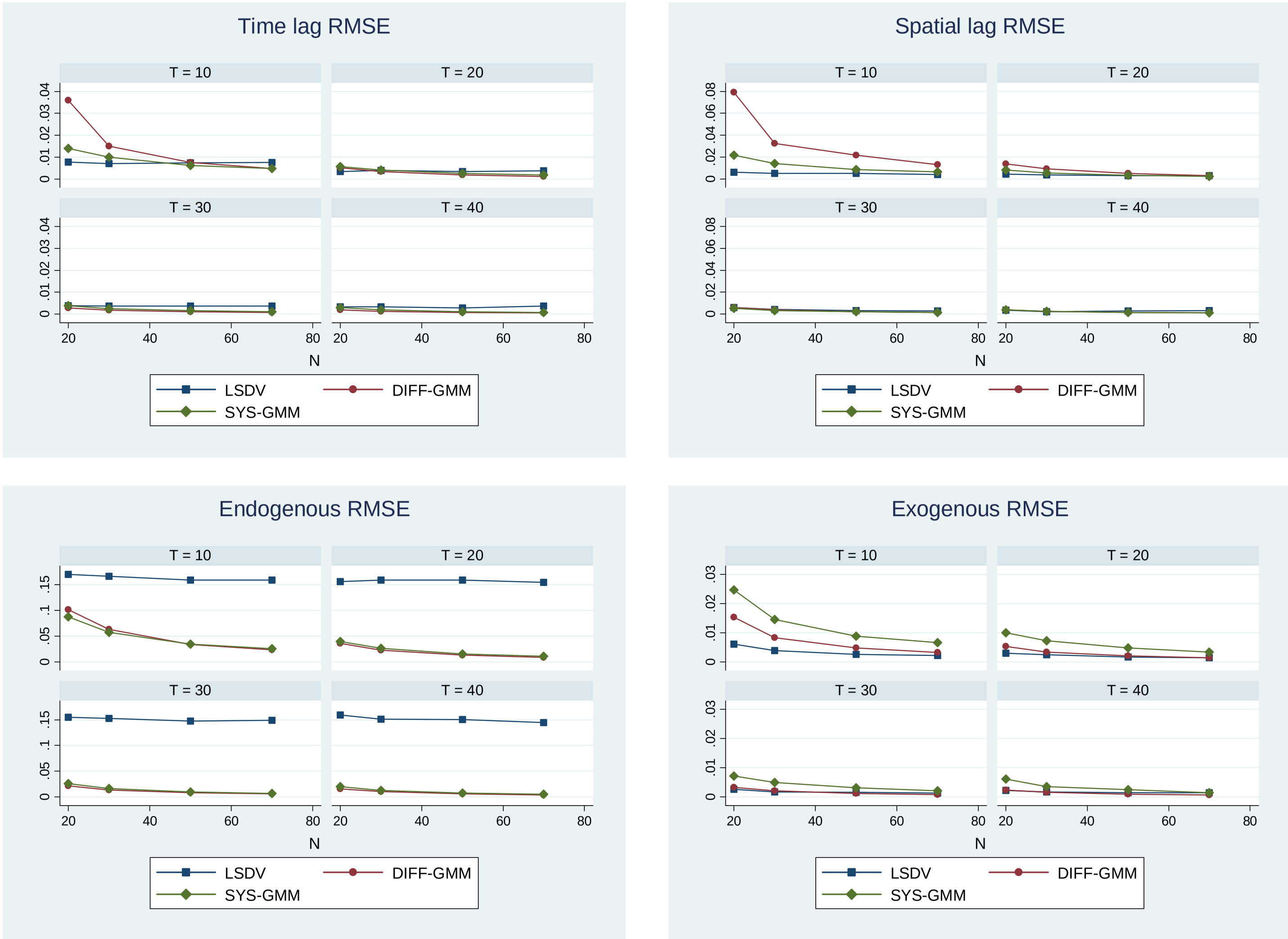

Figure 4: RMSE Extended GMM *vs.* Spatial Estimators

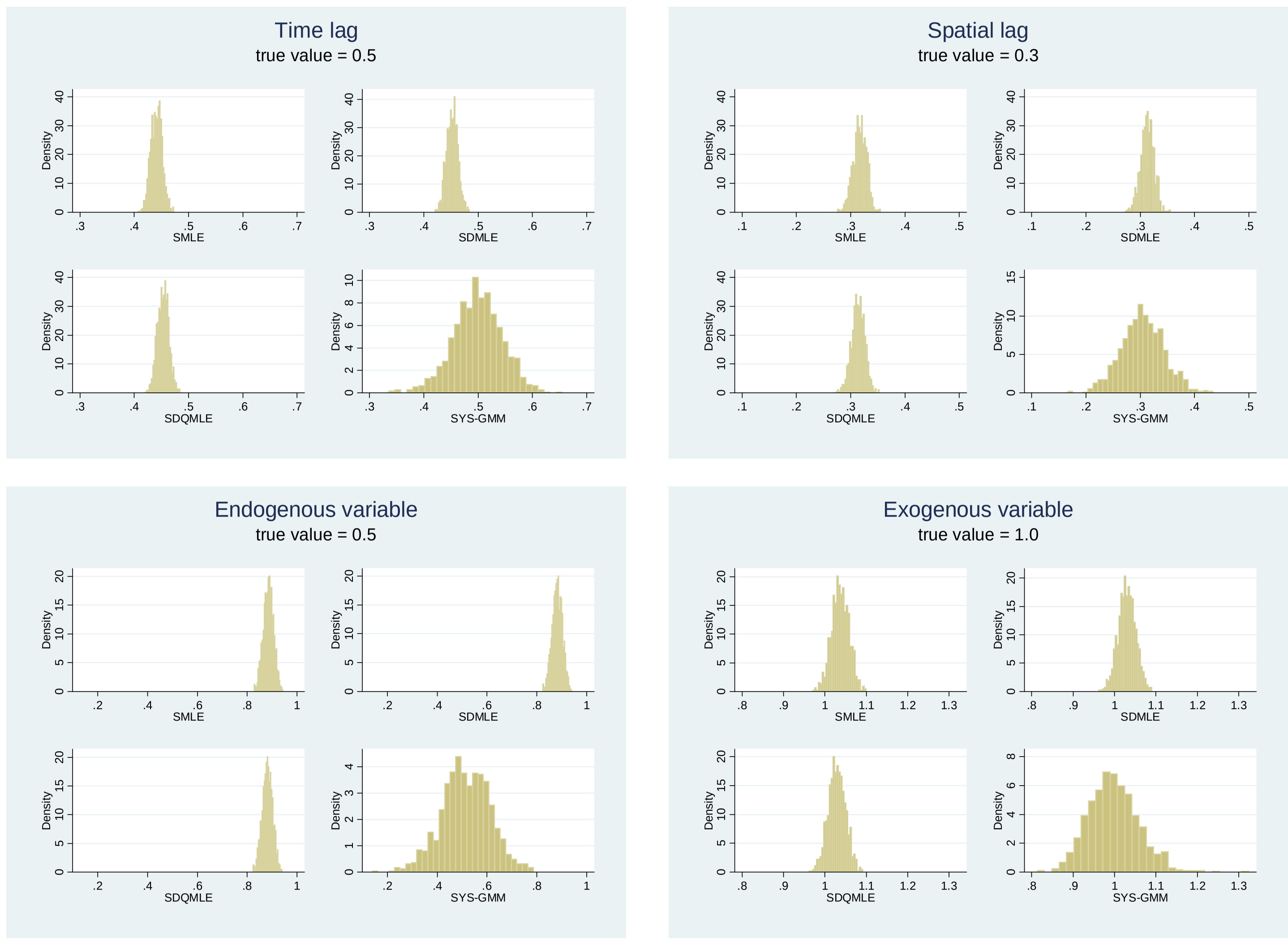

Figure 5: Histograms of the parameters ($N = 70$ and $T = 20$)

4 Robustness Check

In this section, we investigate the sensitivity of our main results. First, we describe the results in terms of RMSE response functions. Second, we check if the results are sensitive to the choice of the spatial matrix W. Additionally, we analyze what happens when the spatial weight scheme is miss-specified. Third, we drop the assumption of Gaussian process for the error terms and individual effects. Last, we investigate the consequence of dealing with spatially dependent endogenous and exogenous variables[16].

4.1 RMSE response function

As previously commented, the relationship between the performance of the estimators and the model parameters is not necessarily easily determined. That is why, we report the results in terms of response functions. Using the RMSE for each estimator and parameters of the entire set of designs, the following equation is estimated by OLS:

$$\begin{aligned}\log\left(\sqrt{N_i \cdot T_i} \cdot RMSE_i\right) &= a_1 + a_2\frac{1}{W_i} + a_3\alpha_i + a_4\rho_i + a_5\left(\alpha_i\rho_i\right) + \\ & a_6\frac{W_i}{N_i} + a_7\frac{W_i}{T_i} + a_8\frac{1}{N_i} + a_9\frac{1}{T_i} + \epsilon_i\end{aligned}$$

where $RMSE_i$ is the RMSE of a given parameter obtained using a given estimator in the i-th design. Note that W_i corresponds to the value attributed to the construction of the spatial weight matrix and is a measure of the degree of sparseness of the matrix W (e.g. $W_i \in (1, 3, 5)$). Note that the dependent variable is expressed in logarithm in order to rule out negative predicted RMSE. Therefore, the interpretation of the estimated coefficients is not straightforward (i.e. a negative coefficient implies a small (close to zero) effect).

The RMSE response function estimation results are displayed in table 2. Although some estimations are affected by multicollinearity, the fit of the response functions to the data is relatively good as suggested by R^2. The comments made in the previous section are confirmed by the estimation results. As expected, the main factor that contributes to the efficiency of the GMM estimator is the panel size (N and T). This is not the case

[16] To conserve space, we dont display the results tables, but they remain available upon request.

Table 2: RMSE Response Functions

	Time lag: α			Spatial lag: ρ			Endogenous variable: γ			Exogenous variable: β		
	SYS-GMM	SDMLE	SDQMLE	SYS-GMM	SDMLE	SDQMLE	SYS-GMM	SDMLE	SDQMLE	SYS-GMM	SDMLE	SDQMLE
$1/W_i$	0.0251	-0.308***	-0.295***	-0.909***	-0.819***	-0.770***	-0.0352	-0.0138	-0.0146	-0.0396	-0.284***	-0.295***
	(-0.057)	(-0.064)	(-0.061)	(-0.081)	(-0.093)	(-0.092)	(-0.049)	(-0.018)	(-0.018)	(-0.063)	(-0.072)	(-0.072)
α_i	-0.229**	-3.520***	-3.821***	-2.887***	-1.858***	-1.800***	-1.002***	-0.195***	-0.187***	0.0625	-1.822***	-1.907***
	(-0.105)	(-0.122)	(-0.115)	(-0.143)	(-0.15)	(-0.151)	(-0.087)	(-0.032)	(-0.032)	(-0.109)	(-0.12)	(-0.121)
ρ_i	-0.736***	-3.072***	-3.131***	-2.615***	-2.156***	-2.012***	-0.500***	-0.157***	-0.159***	-0.441***	-1.911***	-1.899***
	(-0.116)	(-0.159)	(-0.153)	(-0.187)	(-0.205)	(-0.207)	(-0.1)	(-0.038)	(-0.039)	(-0.14)	(-0.147)	(-0.148)
$\alpha_i \rho_i$	1.437***	4.837***	5.041***	0.0916	5.945***	5.876***	0.348	0.146	0.152	0.786*	2.471***	2.380***
	(-0.384)	(-0.436)	(-0.406)	(-0.556)	(-0.646)	(-0.66)	(-0.317)	(-0.121)	(-0.123)	(-0.433)	(-0.461)	(-0.461)
W_i/N_i	0.706**	0.0427	-0.0181	1.454***	1.991***	2.055***	0.393	0.0724	0.0782	0.582	-0.155	-0.182
	(-0.357)	(-0.384)	(-0.374)	(-0.483)	(-0.484)	(-0.495)	(-0.299)	(-0.107)	(-0.11)	(-0.414)	(-0.369)	(-0.363)
W_i/T_i	-0.297*	0.0969	0.147	0.517**	1.366***	1.310***	-0.0852	0.0107	0.0049	-0.322*	-0.211	-0.206
	(-0.166)	(-0.188)	(-0.177)	(-0.24)	(-0.294)	(-0.297)	(-0.137)	(-0.048)	(-0.049)	(-0.186)	(-0.201)	(-0.195)
$1/N_i$	13.40***	-11.79***	-13.13***	13.28***	4.830***	3.885**	16.85***	-16.87***	-17.16***	15.56***	-1.461	-1.483
	(-1.317)	(-1.482)	(-1.41)	(-1.731)	(-1.657)	(-1.691)	(-1.079)	(-0.38)	(-0.391)	(-1.402)	(-1.46)	(-1.444)
$1/T_i$	13.22***	-3.730***	-4.408***	12.70***	1.976**	3.291***	11.04***	-7.673***	-7.513***	9.222***	-3.755***	-3.329***
	(-0.613)	(-0.723)	(-0.669)	(-0.865)	(-0.92)	(-0.939)	(-0.52)	(-0.166)	(-0.173)	(-0.647)	(-0.723)	(-0.711)
Constant	-3.409***	0.202***	0.302***	-0.637***	-3.258***	-3.345***	-1.191***	2.476***	2.474***	-2.762***	-1.206***	-1.228***
	(-0.057)	(-0.072)	(-0.068)	(-0.084)	(-0.09)	(-0.091)	(-0.049)	(-0.02)	(-0.019)	(-0.065)	(-0.072)	(-0.072)
R^2	0.824	0.824	0.856	0.893	0.797	0.791	0.85	0.962	0.961	0.698	0.612	0.618

The standard deviation is reported in parenthesis. ** p<0.01, ** p<0.05, * p<0.1.

for the spatial estimators, because the rate of efficiency is only marginally affected by the panel size. This was already confirmed by the figures 3 and 4, where the slope of the spatial estimators' RMSE were relatively flat. The effects of the autoregressive and spatial autoregressive parameters on the RMSE are clearly non-linear. While the individual effects of the parameters are negative and significant, the interaction term enters the RMSE regression positively and significantly in almost all cases. This result means that it is the combination of the time and spatial lag coefficients which affects the efficiency of the estimators rather than each parameter taken individually. Another important finding relates to the specification of the spatial weight matrix W. The performances of the spatial estimators are definitively more sensitive to the spatial weight matrix than the extended GMM estimator. The spatial lag parameter is the only parameter where performance of GMM seemed to be affected by the spatial matrix W. This finding suggests that spatial estimators are more sensitive to the specification of the spatial weight matrix than GMM. We address this question in the following analysis.

4.2 Role of the Spatial Weight Matrix

In order to check the sensitivity of our results to the choice of spatial matrix, we re-run Monte Carlo experiments for three types of spatial matrices: "3 ahead and 3 behind", "5 ahead and 5 behind" and a negative exponential matrix based on real data distance for 224 countries[17]. As in the baseline case, we perform 1000 simulations for each type of spatial matrix.

The results seems to be qualitatively similar to what we obtained for our baseline case. Extended GMM outperforms the spatial estimators according to unbiasness criteria. The only exception is the spatial lag parameter which is better estimated by spatial ML estimators in relatively small samples. As before, all estimators tend to underestimate autoregressive parameter and overestimate the spatial autoregressive parameter. The coefficients of the endogenous and exogenous variable tend to be overestimated by spatial ML estimators. As before the bias of the endogenous and exogenous variables do not seem to decrease with an increase in the sample size (N and/or T). As highlighted by the response function analysis, the bias of the spatial estimators seems to increase with the number of non-zeros elements in the spatial weight matrix. This is not the case for SYS-GMM, whose performance seems to be not affected by the degree of sparseness of the matrix W. Note that the spatial estimators' sensitivity to the type of spatial weight matrix tends to disappear as the panel dimension increases.

[17] See section 3 for further details.

Most of the spatial econometrics literature presumes that the spatial weight matrix is known and well specified. However, in practice, one has to define the spatial weight matrix. This is usually done based on some underlying theory. But since there is no formal theoretical guidance on the choice of the matrix W, the latter can be misspecified. In order to study the consequences of the misspecification of the spatial weight matrix, we estimate the spatial dynamic panel model using the data generated by the "3 ahead and 3 behind" spatial matrix, but assuming the spatial weight is "1 ahead and 1 behind" and "5 ahead and 5 behind". In the first case, we assume the spatial dependence to be more local than it actually is while in the second scenario we presume the opposite.

The results depend on the type of misspecification. In the case of over-spatial dependence, the results are similar to what we obtained in the baseline case, although the parameters' bias tends to be higher, this is particular true for system GMM. It seems that assuming more global form of spatial dependence introduces additional noise, which affects the performance of extended GMM. As found earlier, the autoregressive variable tends to be underestimated while the spatial autoregressive variable, endogenous and exogenous variables tend to be overestimated. Moreover, the estimation of the autoregressive and endogenous variables by the spatial estimators are dominated by SYS-GMM according to the unbiasness criterion.

In the case of under-spatial dependence, the results for all estimators are severely affected. The bias for the spatial autoregressive parameter changes from negative to positive. In fact, limiting spatial dependence to the close neigbourhood tends to underestimate the spatial effect, although it remains the only parameter which is less affected. The main explanation for this finding lies in the fact that spatial methods estimate the reduced form of the model, which means that the entire set of parameters, beside the spatial lag, are affected by the miss-specification (see equation (18)). As noted previously, simple SMLE seems to estimate the spatial lag as accurate as SDMLE and SDQMLE (Elhorst (2008)).

From an applied econometric point of view, these results suggest that spatial dynamic panel models should not be estimated using only one type of spatial weight matrix. In fact, different types of spatial weight matrices (contiguity, geographical distance, economic distance) should be applied to check the robustness and consistency of the results.

4.3 Non-Gaussian Distribution

Most spatial estimators rely on the assumption of normality of the individual effects and error term. In empirical application, the data does not necessarily follow a normal distribution. That is why we investigate the consequences of dropping the Gaussian assumption through three modifications: student distribution and chi-square distribution for the error term as well as a non-normal distribution for the individual fixed effects.

First, when the error are generated according to a Student distribution with five degrees of freedom, which is characterized by heavier tails than the normal distribution, the results remain qualitatively similar. Extended GMM tends to outperform spatial ML estimators although the rate of convergence seems to be slower. Among the spatial estimators, SDQMLE continues to displays more robustness. This corroborates the finding by Lee (2004) that QMLE can be consistent when the disturbances are independently and identically distributed without normality.

The performance of system GMM deteriorates when the shocks are generated according to a chi-square distribution with 1 degree of freedom. The bias reduction associated with an increase in the panel dimension tends to be smaller than in the baseline case. Spatial ML estimators tend to outperform extended GMM according to the unbiasness criterion for the exogenous and spatial autoregressive parameters However, the estimation of the lagged dependent and endogenous variables with SYS-GMM continues to dominate the spatial ML estimators.

Following Binder et al. (2005), we also investigate the way the individual effects are generated, since the performance of extended-GMM depends on the ratio of the individual effect variance with respect to the variance of the error term (Hayakawa (2006)). Hence, the individual specific effects are no longer normally distributed but generated as follows:

$$\eta_i = \sqrt{\tau}\left(\frac{q_i - 1}{\sqrt{2}}\right) m_i, \qquad q_i \overset{iid}{\backsim} \chi^2(1), \qquad m_i \overset{iid}{\backsim} N(0, 0.05)$$

The parameter τ measures the degree of cross-section to the time-series variations. Two values are considered $\tau = 1$ and $\tau = 5$. As expected, spatial estimators are not affected by the way the individual specific effects are generated. This result is in line with Binder et al.'s (2005) finding. The performance of system GMM deteriorates slightly, but continues to display better results for the endogenous variable than any other spatial estimators.

4.4 Additional Spatial Dependence

Finally, we modify the data generating process to account for spatial dependence in the exogenous and endogenous variables. Each one follows a spatial moving average process (i.e. the shock in one cross-section affects the neighboring cross-sections):

$$\begin{aligned} EX_{it} &= \delta EX_{i,t-1} + \rho_{EX} \left[W u_t \right]_i + u_{it} \\ EN_{it} &= \lambda EN_{i,t-1} + \psi \eta_i + \theta v_{it} + \rho_{EN} \left[W e_t \right]_i + e_{it} \end{aligned}$$

The performance of the spatial estimators and GMM remains relatively robust to the presence of spatial dependence. The relative performance of system GMM with respect to the alternative estimators seems to be unaffected by the introduction of additional spatial dependence through the endogenous and exogenous covariates. Unlike the main results, the spatial ML estimators tend to underestimate the spatial lag parameter, while overestimating the effect of endogenous and exogenous variables. It looks like the spatial estimators attribute the spatial dependence to the endogenous and exogenous variables at the expense of the spatial autoregressive variable. Extended GMM is still less biased and more efficient than the spatial estimators for all variables except the spatial lag variable. The estimation of the spatial lag with SYS-GMM leads to a negative bias, which means that extended GMM continues to overestimate the effect of spatial autoregressive term. In relatively small samples, the spatial estimators continue to dominate system GMM for the spatial lag variable. However, as sample size increases, extended GMM reaches the same level of efficiency as spatial estimators. Note that the bias of exogenous variable for extended GMM tends to be higher compared to the baseline case. Relative performance of extended GMM remains qualitatively unchanged according to RMSE criterion, despite the fact that RMSE tends to be higher for all the estimators in small samples.

Although not reported here, in the presence of positive spatial dependence in the exogenous and endogenous variables, extended GMM performs even better in terms of bias and efficiency, when the spatially weighted sum of the exogenous variable ($W \cdot EX_t$) is included as additional instrument. From a practical viewpoint, this finding suggests to include the spatially weighted average of the exogenous variables as instrument, once the presence of spatial dependence is verified (through a Moran's I or Geary's C test for instance). Note that Kelejian & Prucha (1998) propose to use additional instruments like $W^2 \cdot EX_t$, which can be interpreted as the spatially weighted average of the exogenous variables of the neighborhood's neighborhood.

5 Conclusion

In the presence of endogenous covariates in a spatial dynamic pane framework, the Monte Carlo analysis demonstrates that while the simultaneity bias of the spatial lag remains relatively low, the bias of the endogenous is large if it is not corrected. Proper correction leads to favour extended GMM. In fact, system-GMM emerges clearly dominant by an unbiasedness criterion for most variables, including the endogenous variable. Its RMSE decays at a faster rate as N or T increases and its standard error accuracy is acceptable. In some cases, system GMM can be even more efficient than spatial maximum likelihood estimators. Moreover from a viewpoint purely practical, extended GMM avoids the inversion of a large spatial weight matrix, is easier to implement and its computation time is definitively lower than any maximum likelihood estimators. In addition, the efficiency of the extended GMM could be improved through iterated GMM and continuously updated GMM (Hansen et al. (1996)). Another possibility would be to extend the spatial dynamic MLE and QMLE to a simultaneous equation system framework.

Recently, a lot of attention has been drawn to the impact of heterogenous and cross-section error on the bias in dynamic panel estimation with fixed effects. One way to consider weak cross-section dependence is to allow the error term to be spatially dependent. This could be done by extending the spatial HAC framework proposed by Keleijan & Prucha (2007) or Driscoll & Kray's (1998) approach to the system GMM.

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6 Appendices

6.A GMM Estimators

This appendix section presents the procedure associated with the different GMM estimators. Let Y, Y_{-1}, WY, U be $N \cdot T$ column vectors, EX, is a $N \cdot T \times p$ matrix and EN is a $N \cdot T \times q$ matrix. Note that the data is first sorted by time T and then by cross-section N. Thus, $Y = (Y_1;\ Y_2;\ ...;\ Y_T)'$, where $Y_t = (Y_{1t};\ Y_{2t};\ ...;\ Y_{Nt})'$. The same structure is applied to the remaining vectors and matrices.

As mentioned previously, the time lag ($Y_{i,t-1}$), spatial lag ($[WY_t]_i$) and endogenous variable (EN_{it}) are treated as endogenous covariates, while the exogenous variable (EX_{it}) is considered as strictly exogenous. Each endogenous variable is instrumented by the strictly exogenous variable and the second and third lags of each endogenous variable. In order to restrict the number of instruments, the instruments matrix is constructed applying the "collapse" option[18].

6.A.1 Difference-GMM

The difference-GMM estimator, proposed by Arellano and Bond (1991), consists of estimating the model expressed in first-difference. More specifically, the estimation steps are:

1. Construct the "collapsed" instruments matrix for each cross-section i:

$$Z_i^D = \begin{bmatrix} Y_{i,0} & 0 & [WY_0]_i & 0 & \Delta EX_{i,1} & EN_{i,0} & 0 \\ Y_{i,1} & Y_{i,0} & [WY_1]_i & [WY_0]_i & \Delta EX_{i,2} & EN_{i,1} & EN_{i,0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{i,T-2} & Y_{i,T-3} & [WY_{T-2}]_i & [WY_{T-3}]_i & \Delta EX_{i,T} & EN_{i,T-2} & EN_{i,T-3} \end{bmatrix}$$

2. Construct the weighting matrix:

$$A^{D1} = \left(\sum_i Z_i^{D\prime} \cdot H_i^{D1} \cdot Z_i^D \right)^{-1}$$

where
$$H_i^{D1} = \begin{bmatrix} 1 & -0.5 & 0 & \cdots & 0 \\ -0.5 & 1 & -0.5 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & -0.5 \\ 0 & 0 & 0 & -0.5 & 1 \end{bmatrix}$$

3. Carry out the one-step estimation given by

$$\begin{bmatrix} \widehat{\alpha}_1 \\ \widehat{\rho}_1 \\ \widehat{\beta}_1 \\ \widehat{\gamma}_1 \end{bmatrix} = \left[Q_{XZ} \cdot A^{D1} \cdot Q'_{XZ} \right]^{-1} \cdot Q_{XZ} \cdot A^{D1} \cdot Q_{ZY}$$

where $Q_{XZ} = \sum_i X_i^{*\prime} \cdot Z_i$ and $Q_{ZX} = \sum_i Z_i' \cdot Y_i^*$

$$X_i^* = \begin{bmatrix} \Delta Y_{i,1} & [W\Delta Y_2]_i & \Delta EX_{i,2} & \Delta EN_{i,2} \\ \vdots & \vdots & \vdots & \vdots \\ \Delta Y_{i,T-1} & [W\Delta Y_T]_i & \Delta EX_{i,T} & \Delta EN_{i,T} \end{bmatrix} \text{ and } Y_i^* = \begin{bmatrix} \Delta Y_{i,2} \\ \vdots \\ \Delta Y_{i,T} \end{bmatrix}$$

[18] Numerous instruments can lead to two types of small-sample issues. The first problem leads to overfitting endogenous variables, i.e. failure to remove endogeneity. The second problem concerns imprecise estimation of the optimal weighting matrix in the two-step procedure. This affects the computation of two-step standard errors and the validity of the Hansen's weak instruments (see Roodman (2009)).

4. The associated variance are computed as follows:
$\widehat{V}_1 = \widehat{\sigma}_1^2 \cdot \left[Q_{XZ} \cdot A_i^{D1} \cdot Q'_{XZ}\right]^{-1}$
where $\widehat{\sigma}_1^2 = \frac{1}{N-4} \sum_i \left(Y_i^* - X_i^* \cdot \left[\widehat{\alpha}_1, \widehat{\rho}_1, \widehat{\beta}_1, \widehat{\gamma}_1\right]'\right)' \left(Y_i^* - X_i^* \cdot \left[\widehat{\alpha}_1, \widehat{\rho}_1, \widehat{\beta}_1, \widehat{\gamma}_1\right]'\right)$

5. The robust one-step variance is given by:
$\widehat{V}_{1,Robust} = \left[Q_{XZ} \cdot A^{D1} \cdot Q'_{XZ}\right]^{-1} \cdot A^{D1} \cdot \left(A^{D2}\right)^{-1} \cdot A^{D1} \cdot Q'_{XZ} \cdot \left[Q_{XZ} \cdot A^{D1} \cdot Q'_{XZ}\right]^{-1}$
where $A^{D2} = \left(\sum_i Z_i^{D\prime} \cdot H_i^{D2} \cdot Z_i^D\right)^{-1}$
$H_i^{D2} = \left(Y_i^* - X_i^* \cdot \left[\widehat{\alpha}, \widehat{\rho}, \widehat{\beta}, \widehat{\gamma}\right]'\right) \left(Y_i^* - X_i^* \cdot \left[\widehat{\alpha}, \widehat{\rho}, \widehat{\beta}, \widehat{\gamma}\right]'\right)'$

6. The two-step estimates are given by:
$$\begin{bmatrix} \widehat{\alpha}_2 \\ \widehat{\rho}_2 \\ \widehat{\beta}_2 \\ \widehat{\gamma}_2 \end{bmatrix} = \left[Q_{XZ} \cdot A^{D2} \cdot Q'_{XZ}\right]^{-1} \cdot Q_{XZ} \cdot A^{D2} \cdot Q_{ZY}$$

7. The associated two-step variance is computed as:
$\widehat{V}_2 = \left[Q_{XZ} \cdot A^{D2} \cdot Q'_{XZ}\right]^{-1}$

6.A.2 Extended-GMM

The system-GMM estimator, proposed by Arellano and Bover (1995) and Blundell and Bond (1998), consists of combining the moment conditions from the model in first-difference with the moment conditions from the model in levels. These are the estimation steps:

1. Construct the "collapsed" instruments matrix for each cross-section i:

$$Z_i = \left[\begin{array}{cccccccccc}
Y_{i,0} & 0 & 0 & [WY_0]_i & 0 & 0 & \Delta EX_{i,1} & EN_{i,0} & 0 & 0 \\
Y_{i,1} & Y_{i,0} & 0 & [WY_1]_i & [WY_0]_i & 0 & \Delta EX_{i,2} & EN_{i,1} & EN_{i,0} & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
Y_{i,T-2} & Y_{i,T-3} & 0 & [WY_{T-2}]_i & [WY_{T-3}]_i & 0 & \Delta EX_{i,T} & EN_{i,T-2} & EN_{i,T-3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & EX_{i1} & 0 & 0 & 0 \\
0 & 0 & \Delta Y_{i,1} & 0 & 0 & [W\Delta Y_2]_i & EX_{i2} & 0 & 0 & \Delta EN_{i2} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \Delta Y_{i,T-1} & 0 & 0 & [W\Delta Y_{T-1}]_i & EX_{iT} & 0 & 0 & \Delta EN_{i,T-1}
\end{array}\right]$$

2. Construct the weighting matrix:

$$A^1 = \left(\sum_i Z_i' \cdot H_i \cdot Z_i \right)^{-1}$$

where $H_i = \begin{bmatrix} H_i^D & 0 \\ 0 & I_i \end{bmatrix}$

3. Carry out the one-step estimation given by:

$$\begin{bmatrix} \widehat{\alpha}_1 \\ \widehat{\rho}_1 \\ \widehat{\beta}_1 \\ \widehat{\gamma}_1 \end{bmatrix} = \left[Q_{XZ} \cdot A^1 \cdot Q_{XZ}' \right]^{-1} \cdot Q_{XZ} \cdot A^1 \cdot Q_{ZY}$$

where $Q_{XZ} = \sum_i X_i^{*\prime} \cdot Z_i$ and $Q_{ZX} = \sum_i Z_i' \cdot Y_i^*$

$$X_i^* = \begin{bmatrix} \Delta Y_{i,1} & [W\Delta Y_2]_i & \Delta EX_{i,2} & \Delta EN_{i,2} \\ \vdots & \vdots & \vdots & \vdots \\ \Delta Y_{i,T-1} & [W\Delta Y_T]_i & \Delta EX_{i,T} & \Delta EN_{i,T} \\ Y_{i,0} & [WY_1]_i & EX_{i,1} & EN_{i,1} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{i,T-1} & [WY_T]_i & EX_{i,T} & EN_{i,T} \end{bmatrix} \text{ and } Y_i^* = \begin{bmatrix} \Delta Y_{i,2} \\ \vdots \\ \Delta Y_{i,T} \\ Y_{i,1} \\ \vdots \\ Y_{i,T} \end{bmatrix}$$

4. The associated variance are computed as follows:

$$\widehat{V}_1 = \widehat{\sigma}_1^2 \cdot \left[Q_{XZ} \cdot A_i^1 \cdot Q_{XZ}' \right]^{-1}$$

where $\widehat{\sigma}_1^2 = \frac{1}{N-4} \sum_i \left(Y_i^* - X_i^* \cdot \left[\widehat{\alpha}_1, \widehat{\rho}_1, \widehat{\beta}_1, \widehat{\gamma}_1 \right]' \right)' \left(Y_i^* - X_i^* \cdot \left[\widehat{\alpha}_1, \widehat{\rho}_1, \widehat{\beta}_1, \widehat{\gamma}_1 \right]' \right)$

5. The robust one-step variance is given by:

$$\widehat{V}_{1,Robust} = \left[Q_{XZ} \cdot A^1 \cdot Q_{XZ}' \right]^{-1} \cdot A^1 \cdot (A^2)^{-1} \cdot A^1 \cdot Q_{XZ}' \cdot \left[Q_{XZ} \cdot A^1 \cdot Q_{XZ}' \right]^{-1}$$

where $A^2 = \left(\sum_i Z_i' \cdot H_i^2 \cdot Z_i \right)^{-1}$

$$H_i^2 = \left(Y_i^* - X_i^* \cdot \left[\widehat{\alpha}, \widehat{\rho}, \widehat{\beta}, \widehat{\gamma} \right]' \right) \left(Y_i^* - X_i^* \cdot \left[\widehat{\alpha}, \widehat{\rho}, \widehat{\beta}, \widehat{\gamma} \right]' \right)'$$

6. The two-step estimates are given by:

$$\begin{bmatrix} \widehat{\alpha} \\ \widehat{\rho} \\ \widehat{\beta} \\ \widehat{\gamma} \end{bmatrix} = \left[Q_{XZ} \cdot A^2 \cdot Q_{XZ}' \right]^{-1} \cdot Q_{XZ} \cdot A^2 \cdot Q_{ZY}$$

7. The associated two-step variance is computed as:

$$\widehat{V}_2 = \left[Q_{XZ} \cdot A^2 \cdot Q_{XZ}' \right]^{-1}$$

6.B Spatial Estimators

This appendix section presents the procedure associated with the different spatial estimators. For further details, the reader is referred to Anselin (1988), Elhorst (2003a, 2005, 2008) and Yu et al. (2008). Let Y, Y_{-1}, WY, U be $N \cdot T$ column vectors, EX,is a $N \cdot T \times p$ matrix and EN is a $N \cdot T \times q$ matrix. Note that the data is first sorted by time T and then by cross-section N. Thus, $Y = (Y_1;\ Y_2;\ ...;\ Y_T)'$, where $Y_t = (Y_{1t};\ Y_{2t};\ ...;\ Y_{Nt})'$. The same structure is applied to the remaining vectors and matrices. As initial values for the parameters, the estimates obtained by extended-GMM can be used.

6.B.1 Spatial MLE

The classical spatial maximum likelihood estimator relies on the concentrated likelihood in the spatial lag parameter, which is conditional upon the others' coefficient values. Operationally, "standard" spatial maximum estimation can be achieved in five steps:

1. Demean all variables, denoted by $\tilde{}$.
2. Carry out the following OLS regressions:
   $\widetilde{Y} = \left[\widetilde{Y}_{-1}; \widetilde{EX}; \widetilde{EN}\right] b_0 + U_0$
   $W\widetilde{Y} = \left[\widetilde{Y}_{-1}; \widetilde{EX}; \widetilde{EN}\right] b_L + U_L.$
3. Compute the associated residuals $\widehat{U}_0$ and $\widehat{U}_L$.
4. Given $\widehat{U}_0$ and $\widehat{U}_L$, find ρ that maximizes the following concentrated likelihood:
   $\ln L(\rho) = -\frac{NT}{2}\ln 2\pi - \frac{NT}{2}\ln\sigma^2 + T\ln|I_N - \rho W| - \frac{NT}{2}\ln\left[\left(\widehat{U}_0 - \rho\widehat{U}_L\right)'\left(\widehat{U}_0 - \rho\widehat{U}_L\right)\right].$
5. Given the estimate $\widehat{\rho}$, the remaining coefficient estimates are computed as follows:
   $\begin{bmatrix} \widehat{\alpha} \\ \widehat{\beta} \\ \widehat{\gamma} \end{bmatrix} = b_0 - \widehat{\rho} b_L \qquad \text{and} \qquad \widehat{\sigma}^2 = \frac{1}{NT}\left(\widehat{U}_0 - \widehat{\rho}\widehat{U}_L\right)'\left(\widehat{U}_0 - \widehat{\rho}\widehat{U}_L\right).$

As mentioned in Elhorst (2008), this spatial MLE is inconsistent, because of the presence of the lag dependent variable.

6.B.2 Spatial Dynamic MLE

The unconditional MLE, proposed by Elhorst (2005, 2008), involves a two-steps iterative procedure once the data has been first-differenced. Note that the initial observations are approximated using Bhargava and Sargan approach (1983). Estimation should proceed according to the following steps:

1. Take the first-difference of all variables.
2. Define some initial values for the parameters α, ρ and θ, where $\theta = \sigma_\xi^2/\sigma^2$ and σ_ξ^2 is the variance associated with the approximation of the initial observations.

3. The two-steps iterative procedure begins here with the computation of the coefficients π_i associated with the initial observations's approximation as well as the parameters of the exogenous and endogenous covariates, and the variance σ^2 :

$$\begin{bmatrix} \widehat{\pi}_1 \\ \widehat{\pi}_2 \\ \vdots \\ \widehat{\pi}_T \\ \widehat{\beta} \\ \widehat{\gamma} \end{bmatrix} = \left(\underline{\Delta X}' H_{V\theta}^{-1} \underline{\Delta X}\right)^{-1} \underline{\Delta X}' H_{V\theta}^{-1} \underline{\Delta Y} \qquad \text{and} \qquad \widehat{\sigma}^2 = \frac{\Delta \widehat{U}' H_{V\theta}^{-1} \Delta \widehat{U}}{NT}$$

$$\text{where} \quad \underline{\Delta X} = \begin{bmatrix} \imath_N & \Delta X_1 & \cdots & \Delta X_T & 0 \\ 0 & 0 & \cdots & 0 & \Delta X_2 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \Delta X_T \end{bmatrix};$$

$$\underline{\Delta Y} = \begin{bmatrix} (I_N - \rho W)\, \Delta Y_1 \\ (I_N - \rho W)\, \Delta Y_2 - \alpha \Delta Y_1 \\ \vdots \\ (I_N - \rho W)\, \Delta Y_T - \alpha \Delta Y_{T-1} \end{bmatrix};$$

$$H_{V\theta} \begin{bmatrix} V_\theta & -I_N & 0 & \cdots & 0 & 0 \\ -I_N & 2 \cdot I_N & -I_N & \ddots & 0 & 0 \\ 0 & -I_N & 2 \cdot I_N & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 \cdot I_N & -I_N \\ 0 & 0 & 0 & \cdots & -I_N & 2 \cdot I_N \end{bmatrix};$$

$$\begin{aligned} V_\theta = \theta I_N + I_N &+ (\alpha S - I_N) \left(I_N - \alpha^2 S S'\right)^{-1} (\alpha S - I_N)' \\ &- (\alpha S - I_N) (\alpha S)^{m-1} \left(I_N - \alpha^2 S S'\right)^{-1} (\alpha S)^{m-1} (\alpha S - I_N)' \\ &- \left(\alpha^2 S S'\right)^{m-1}; \end{aligned}$$

$$S = (I_N - \rho W)^{-1};$$

$$\Delta \widehat{U} = \underline{\Delta Y} - \underline{\Delta X} \cdot \left(\widehat{\pi}_1;\ ...;\ \widehat{\pi}_T;\ \widehat{\beta}';\ \widehat{\gamma}'\right);$$

   The parameter m, which represents the number of periods since the process started, should be defined in advance. It must be such that the eigenvalues of the matrix αS lie inside the unit circle, because otherwise the matrix $(\alpha S)^{m-1}$ would become infinite and yield a corner solution. Elhorst (2008) proposes to include a third step procedure to estimate m. Beside increasing the computation time, this additional step improves marginally the results.

4. Given the set of parameters obtained in step 3, maximize the unconditional likelihood function as follows:

$$\ln L(\alpha, \rho, \theta) = -\frac{NT}{2} \ln 2\pi - \frac{NT}{2} \ln \sigma^2 + T \ln |I_N - \rho W| - \frac{1}{2} \ln |H_{V\theta}| - \frac{1}{2\sigma^2} \Delta \widehat{U}' H_{V\theta}^{-1} \Delta \widehat{U}$$

   w.r.t. $|\alpha| < 1 - \rho\omega_{\max}$ and $|\alpha| < 1 - \rho\omega_{\min}$

5. Repeat step 3, with the estimates obtained in step 4 and so on.., until convergence is met.

Note that to reduce the computation time the jacobian term, $\ln |I_N - \rho W|$, in the loglikelihood function is approximation by $\sum_{i=1}^{N} \ln (1 - \rho\omega_i)$, where ω_i is the eigenvalue of the matrix W. The inverse of matrix $H_{V\theta}$ is also estimated using summation operations instead of matrix calculus.

6.B.3 Spatial Dynamic QMLE

The QMLE, presented by Yu et al. (2008), requires first the maximization of the concentrated likelihood and then a bias correction. Note that the original model proposed by the authors includes a lagged spatial lag and corresponds to a "time-space dynamic" model (based on Anselin taxonomy (1988, 2001)). The estimation process involves the following steps:

1. Demean all variables, denoted by $\tilde{}$.
2. Maximize the following concentrated likelihood function in order to estimate $\widehat{\alpha}$, $\widehat{\rho}$, $\widehat{\beta}$, $\widehat{\gamma}$ and $\widehat{\sigma}^2$:

   $$\ln L\left(\alpha,\rho,\beta,\gamma,\sigma^2\right) = -\frac{NT}{2}\ln 2\pi - \frac{NT}{2}\ln\sigma^2 + T\ln\left|I_N-\rho W\right| - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\tilde{U}_t'\tilde{U}_t$$

   $$\text{w.r.t.}\quad \begin{aligned} &\sum_{t=1}^{T}\widetilde{Y}_{-1}'\tilde{U}_t = 0\\ &\sum_{t=1}^{T}\left(W\widetilde{Y}_{-1}'\right)'\tilde{U}_t = tr\left(W\left(I_N-\rho W\right)^{-1}\right)\\ &\sum_{t=1}^{T}\widetilde{EX}'\tilde{U}_t = 0\\ &\sum_{t=1}^{T}\widetilde{EN}'\tilde{U}_t = 0\\ &\sum_{t=1}^{T}\tilde{U}_t'\tilde{U}_t = N\sigma^2 \end{aligned}$$

   where $\tilde{U}_t = \left(I_N-\rho W\right)\widetilde{Y}_t - \left[\widetilde{Y}_{-1};\widetilde{EX};\widetilde{EN}\right]\left[\alpha;\ \beta';\ \gamma'\right]'$

3. The bias-corrected estimator is then given by:

   $$\begin{bmatrix}\widehat{\alpha}^c\\ \widehat{\rho}^c\\ \widehat{\beta}^c\\ \widehat{\gamma}^c\\ \widehat{\sigma}^{2c}\end{bmatrix} = \begin{bmatrix}\widehat{\alpha}\\ \widehat{\rho}\\ \widehat{\beta}\\ \widehat{\gamma}\\ \widehat{\sigma}^2\end{bmatrix} - \frac{1}{T}\left(-\widehat{\Sigma}^{-1}b\right)$$

   where $\widehat{\Sigma}^{-1}$ can be approximated by the empirical Hessian matrix of the concentrated log likelihood function (an analytical expression for the matrix Σ can also be found in Yu et al.) and the column matrix b is given by:

   $$b = \begin{bmatrix} \frac{1}{N}tr\left(\left(I_N-\widehat{\alpha}\left(I_N-\widehat{\rho}W\right)^{-1}\right)\left(I_N-\widehat{\rho}W\right)^{-1}\right)\\ \frac{\widehat{\alpha}}{N}tr\left(W\left(I_N-\widehat{\rho}W\right)^{-1}\left(I_N-\widehat{\alpha}\left(I_N-\widehat{\rho}W\right)^{-1}\right)\left(I_N-\widehat{\rho}W\right)^{-1}\right)+\frac{1}{N}tr\left(W\left(I_N-\widehat{\rho}W\right)^{-1}\right)\\ 0\\ 0\\ \frac{1}{2\widehat{\sigma}^2}\end{bmatrix}$$

4. Finally, the individual effects are recovered as follows:

   $$\widehat{\eta} = \frac{1}{T}\sum_{t=1}^{T}\left(I_N-\widehat{\rho}^cW\right)Y_t - \left[Y_{-1};EX;EN\right]\left[\widehat{\alpha}^c;\ \widehat{\beta}^{c\prime};\ \widehat{\gamma}^{c\prime}\right]'$$

6.C Monte Carlo Results: Bias

Time lag variable α: Bias

T	N	α	ρ	SMLE	SDMLE	SDQMLE	LSDV	DIF-GMM	SYS-GMM
10	20	0.2	0.1	0.1076	0.0860	0.0807	0.0935	0.0588	0.0272
10	20	0.5	0.3	0.0784	0.0633	0.0518	0.0618	0.0472	0.0244
10	20	0.7	0.1	0.0722	0.0562	0.0402	0.0455	0.0762	-0.0561
20	20	0.2	0.1	0.0913	0.0804	0.0784	0.0413	0.0166	0.0180
20	20	0.5	0.3	0.0585	0.0514	0.0467	0.0614	0.0226	0.0115
20	20	0.7	0.1	0.0478	0.0407	0.0352	0.0211	0.0200	-0.0052
30	20	0.2	0.1	0.0857	0.0783	0.0770	-0.0591	0.0122	0.0141
30	20	0.5	0.3	0.0531	0.0483	0.0455	0.0608	0.0141	0.0099
30	20	0.7	0.1	0.0404	0.0362	0.0328	-0.0134	0.0102	-0.0008
40	20	0.2	0.1	0.0819	0.0763	0.0756	0.0803	0.0082	0.0096
40	20	0.5	0.3	0.0500	0.0463	0.0443	-0.0666	0.0081	0.0046
40	20	0.7	0.1	0.0374	0.0343	0.0327	-0.0064	0.0082	0.0004
10	30	0.2	0.1	0.1081	0.0851	0.0821	0.1066	0.0298	0.0284
10	30	0.5	0.3	0.0780	0.0571	0.0506	-0.0066	0.0437	0.0013
10	30	0.7	0.1	0.0716	0.0459	0.0388	0.0651	0.0485	-0.0042
20	30	0.2	0.1	0.0917	0.0807	0.0785	0.0921	0.0122	0.0143
20	30	0.5	0.3	0.0588	0.0514	0.0464	0.0689	0.0148	0.0038
20	30	0.7	0.1	0.0471	0.0404	0.0344	0.0068	0.0119	-0.0082
30	30	0.2	0.1	0.0850	0.0775	0.0763	-0.0514	0.0100	0.0071
30	30	0.5	0.3	0.0525	0.0477	0.0447	0.0345	0.0090	0.0028
30	30	0.7	0.1	0.0398	0.0355	0.0327	-0.0119	0.0081	-0.0059
40	30	0.2	0.1	0.0828	0.0773	0.0762	0.0651	0.0057	0.0059
40	30	0.5	0.3	0.0486	0.0452	0.0432	0.0262	0.0047	0.0044
40	30	0.7	0.1	0.0366	0.0335	0.0320	0.0119	0.0047	-0.0042
10	50	0.2	0.1	0.1070	0.0825	0.0808	0.1064	0.0203	0.0177
10	50	0.5	0.3	0.0795	0.0547	0.0524	0.0601	0.0326	0.0004
10	50	0.7	0.1	0.0706	0.0437	0.0385	0.0589	0.0228	-0.0163
20	50	0.2	0.1	0.0897	0.0786	0.0768	0.0849	0.0091	0.0095
20	50	0.5	0.3	0.0580	0.0492	0.0461	0.0693	0.0087	0.0036
20	50	0.7	0.1	0.0473	0.0377	0.0344	0.0457	0.0090	-0.0106
30	50	0.2	0.1	0.0853	0.0780	0.0767	0.0488	0.0048	0.0051
30	50	0.5	0.3	0.0530	0.0483	0.0453	0.0361	0.0060	0.0007
30	50	0.7	0.1	0.0401	0.0358	0.0327	0.0194	0.0052	-0.0042
40	50	0.2	0.1	0.0818	0.0763	0.0755	0.0588	0.0032	0.0051
40	50	0.5	0.3	0.0499	0.0464	0.0443	0.0596	0.0040	0.0006
40	50	0.7	0.1	0.0369	0.0337	0.0323	-0.0440	0.0044	-0.0051
10	70	0.2	0.1	0.1064	0.0812	0.0802	0.0975	0.0149	0.0136
10	70	0.5	0.3	0.0787	0.0538	0.0515	0.0907	0.0200	0.0048
10	70	0.7	0.1	0.0692	0.0429	0.0375	0.0363	0.0193	-0.0129
20	70	0.2	0.1	0.0907	0.0789	0.0776	0.0910	0.0063	0.0077
20	70	0.5	0.3	0.0583	0.0473	0.0464	0.0628	0.0076	0.0003
20	70	0.7	0.1	0.0468	0.0350	0.0338	0.0352	0.0077	-0.0094
30	70	0.2	0.1	0.0847	0.0773	0.0761	0.0714	0.0034	0.0038
30	70	0.5	0.3	0.0516	0.0461	0.0440	0.0247	0.0036	0.0000
30	70	0.7	0.1	0.0392	0.0336	0.0314	0.0212	0.0041	-0.0025
40	70	0.2	0.1	0.0825	0.0770	0.0760	0.0612	0.0013	0.0020
40	70	0.5	0.3	0.0492	0.0456	0.0434	0.0597	0.0026	0.0019
40	70	0.7	0.1	0.0366	0.0335	0.0309	0.0322	0.0016	-0.0034

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Spatial lag variable ρ: Bias

T	N	α	ρ	SMLE	SDMLE	SDQMLE	LSDV	DIF-GMM	SYS-GMM
10	20	0.2	0.1	-0.0013	-0.0006	0.0005	-0.0825	-0.0148	-0.0326
10	20	0.5	0.3	-0.0151	-0.0116	-0.0104	-0.0883	-0.0216	-0.0140
10	20	0.7	0.1	-0.0081	-0.0085	-0.0073	-0.0812	-0.0253	-0.0005
20	20	0.2	0.1	-0.0054	-0.0053	-0.0048	-0.0460	-0.0127	0.0020
20	20	0.5	0.3	-0.0152	-0.0130	-0.0124	-0.0482	-0.0125	-0.0164
20	20	0.7	0.1	-0.0073	-0.0066	-0.0065	-0.0665	-0.0036	-0.0009
30	20	0.2	0.1	-0.0045	-0.0042	-0.0041	-0.1479	0.0000	-0.0084
30	20	0.5	0.3	-0.0174	-0.0158	-0.0153	-0.0248	-0.0066	-0.0092
30	20	0.7	0.1	-0.0072	-0.0066	-0.0070	0.0752	-0.0035	0.0063
40	20	0.2	0.1	-0.0046	-0.0041	-0.0043	-0.0222	-0.0014	-0.0046
40	20	0.5	0.3	-0.0161	-0.0151	-0.0147	-0.0150	0.0003	-0.0024
40	20	0.7	0.1	-0.0070	-0.0062	-0.0070	-0.0163	-0.0015	-0.0021
10	30	0.2	0.1	-0.0027	-0.0015	-0.0015	-0.0176	-0.0111	-0.0078
10	30	0.5	0.3	-0.0150	-0.0113	-0.0120	-0.0462	-0.0139	-0.0039
10	30	0.7	0.1	-0.0074	-0.0053	-0.0077	-0.0067	-0.0008	-0.0016
20	30	0.2	0.1	-0.0039	-0.0033	-0.0031	-0.0286	-0.0075	-0.0037
20	30	0.5	0.3	-0.0170	-0.0146	-0.0140	-0.0518	-0.0013	-0.0051
20	30	0.7	0.1	-0.0077	-0.0067	-0.0069	0.0040	-0.0014	-0.0056
30	30	0.2	0.1	-0.0044	-0.0039	-0.0041	0.0948	-0.0056	-0.0016
30	30	0.5	0.3	-0.0173	-0.0158	-0.0153	0.0060	0.0017	-0.0029
30	30	0.7	0.1	-0.0068	-0.0059	-0.0060	0.0032	-0.0024	-0.0026
40	30	0.2	0.1	-0.0037	-0.0034	-0.0033	0.0070	0.0032	-0.0026
40	30	0.5	0.3	-0.0160	-0.0151	-0.0147	-0.0202	-0.0046	-0.0056
40	30	0.7	0.1	-0.0081	-0.0076	-0.0078	0.0032	-0.0014	-0.0010
10	50	0.2	0.1	-0.0034	-0.0010	-0.0025	-0.0267	-0.0077	-0.0051
10	50	0.5	0.3	-0.0144	-0.0089	-0.0101	-0.1200	-0.0052	-0.0219
10	50	0.7	0.1	-0.0063	-0.0040	-0.0061	-0.0286	-0.0005	-0.0069
20	50	0.2	0.1	-0.0032	-0.0028	-0.0026	-0.0195	-0.0008	0.0011
20	50	0.5	0.3	-0.0172	-0.0144	-0.0143	-0.0499	0.0001	-0.0035
20	50	0.7	0.1	-0.0079	-0.0060	-0.0068	-0.0219	-0.0005	-0.0027
30	50	0.2	0.1	-0.0039	-0.0034	-0.0035	-0.0522	0.0007	-0.0006
30	50	0.5	0.3	-0.0174	-0.0158	-0.0155	-0.0006	-0.0014	-0.0035
30	50	0.7	0.1	-0.0074	-0.0066	-0.0066	0.0060	-0.0037	-0.0023
40	50	0.2	0.1	-0.0036	-0.0034	-0.0033	-0.0448	0.0037	0.0043
40	50	0.5	0.3	-0.0173	-0.0161	-0.0158	-0.0518	-0.0018	-0.0037
40	50	0.7	0.1	-0.0075	-0.0068	-0.0072	-0.0200	-0.0027	-0.0034
10	70	0.2	0.1	-0.0041	-0.0024	-0.0031	-0.0287	-0.0043	-0.0032
10	70	0.5	0.3	-0.0159	-0.0104	-0.0111	-0.0550	-0.0075	-0.0154
10	70	0.7	0.1	-0.0064	-0.0038	-0.0056	0.0101	0.0027	-0.0039
20	70	0.2	0.1	-0.0044	-0.0038	-0.0038	-0.0230	-0.0033	-0.0029
20	70	0.5	0.3	-0.0176	-0.0134	-0.0144	-0.0443	-0.0006	-0.0037
20	70	0.7	0.1	-0.0077	-0.0058	-0.0068	-0.0081	-0.0017	-0.0034
30	70	0.2	0.1	-0.0045	-0.0040	-0.0041	-0.0114	0.0011	-0.0010
30	70	0.5	0.3	-0.0165	-0.0145	-0.0144	-0.0140	-0.0008	-0.0025
30	70	0.7	0.1	-0.0074	-0.0063	-0.0067	0.0131	0.0004	0.0003
40	70	0.2	0.1	-0.0043	-0.0040	-0.0039	-0.0302	-0.0019	-0.0026
40	70	0.5	0.3	-0.0171	-0.0163	-0.0158	-0.0557	-0.0003	-0.0022
40	70	0.7	0.1	-0.0078	-0.0072	-0.0073	-0.0091	0.0002	-0.0004

Endogenous variable γ: Bias

T	N	α	ρ	SMLE	SDMLE	SDQMLE	LSDV	DIF-GMM	SYS-GMM
10	20	0.2	0.1	-0.4125	-0.4065	-0.4050	-0.4107	-0.1361	-0.1614
10	20	0.5	0.3	-0.3994	-0.3958	-0.3932	-0.4003	-0.1305	-0.1308
10	20	0.7	0.1	-0.3942	-0.3931	-0.3905	-0.4014	-0.1193	-0.0894
20	20	0.2	0.1	-0.3987	-0.3950	-0.3941	-0.4117	-0.0589	-0.0822
20	20	0.5	0.3	-0.3845	-0.3826	-0.3808	-0.3863	-0.0617	-0.0695
20	20	0.7	0.1	-0.3817	-0.3793	-0.3776	-0.3870	-0.0404	-0.0389
30	20	0.2	0.1	-0.3932	-0.3905	-0.3898	-0.4266	-0.0366	-0.0472
30	20	0.5	0.3	-0.3786	-0.3766	-0.3758	-0.3838	-0.0384	-0.0388
30	20	0.7	0.1	-0.3779	-0.3761	-0.3741	-0.3989	-0.0451	-0.0341
40	20	0.2	0.1	-0.3907	-0.3885	-0.3882	-0.3906	-0.0334	-0.0455
40	20	0.5	0.3	-0.3790	-0.3775	-0.3766	-0.4267	-0.0213	-0.0332
40	20	0.7	0.1	-0.3733	-0.3714	-0.3701	-0.3862	-0.0208	-0.0252
10	30	0.2	0.1	-0.4192	-0.4138	-0.4131	-0.4192	-0.0764	-0.1107
10	30	0.5	0.3	-0.3975	-0.3943	-0.3924	-0.4378	-0.0808	-0.0781
10	30	0.7	0.1	-0.3947	-0.3902	-0.3909	-0.3982	-0.0877	-0.0805
20	30	0.2	0.1	-0.3980	-0.3941	-0.3936	-0.3970	-0.0341	-0.0552
20	30	0.5	0.3	-0.3861	-0.3837	-0.3820	-0.3838	-0.0448	-0.0475
20	30	0.7	0.1	-0.3806	-0.3787	-0.3771	-0.3946	-0.0390	-0.0323
30	30	0.2	0.1	-0.3925	-0.3901	-0.3893	-0.4420	-0.0180	-0.0327
30	30	0.5	0.3	-0.3802	-0.3785	-0.3776	-0.3999	-0.0270	-0.0256
30	30	0.7	0.1	-0.3780	-0.3764	-0.3749	-0.3952	-0.0248	-0.0189
40	30	0.2	0.1	-0.3918	-0.3896	-0.3892	-0.3983	-0.0221	-0.0325
40	30	0.5	0.3	-0.3771	-0.3757	-0.3748	-0.3924	-0.0193	-0.0272
40	30	0.7	0.1	-0.3759	-0.3748	-0.3730	-0.3859	-0.0143	-0.0168
10	50	0.2	0.1	-0.4095	-0.4031	-0.4026	-0.4076	-0.0540	-0.0762
10	50	0.5	0.3	-0.3980	-0.3939	-0.3927	-0.3930	-0.0598	-0.0483
10	50	0.7	0.1	-0.3945	-0.3906	-0.3898	-0.3978	-0.0490	-0.0444
20	50	0.2	0.1	-0.3990	-0.3951	-0.3945	-0.3999	-0.0247	-0.0440
20	50	0.5	0.3	-0.3856	-0.3828	-0.3816	-0.3838	-0.0277	-0.0333
20	50	0.7	0.1	-0.3811	-0.3784	-0.3773	-0.3814	-0.0210	-0.0119
30	50	0.2	0.1	-0.3946	-0.3918	-0.3914	-0.4024	-0.0133	-0.0228
30	50	0.5	0.3	-0.3805	-0.3789	-0.3777	-0.3984	-0.0148	-0.0142
30	50	0.7	0.1	-0.3762	-0.3744	-0.3729	-0.3833	-0.0127	-0.0123
40	50	0.2	0.1	-0.3918	-0.3895	-0.3891	-0.3964	-0.0155	-0.0257
40	50	0.5	0.3	-0.3778	-0.3764	-0.3757	-0.3773	-0.0137	-0.0134
40	50	0.7	0.1	-0.3753	-0.3741	-0.3726	-0.3992	-0.0093	-0.0056
10	70	0.2	0.1	-0.4105	-0.4037	-0.4032	-0.4130	-0.0448	-0.0558
10	70	0.5	0.3	-0.4009	-0.3955	-0.3952	-0.3969	-0.0471	-0.0408
10	70	0.7	0.1	-0.3918	-0.3877	-0.3871	-0.4046	-0.0400	-0.0271
20	70	0.2	0.1	-0.3982	-0.3941	-0.3937	-0.3975	-0.0174	-0.0300
20	70	0.5	0.3	-0.3862	-0.3829	-0.3823	-0.3886	-0.0224	-0.0155
20	70	0.7	0.1	-0.3809	-0.3773	-0.3769	-0.3850	-0.0223	-0.0184
30	70	0.2	0.1	-0.3940	-0.3911	-0.3907	-0.3985	-0.0152	-0.0228
30	70	0.5	0.3	-0.3804	-0.3784	-0.3775	-0.3998	-0.0122	-0.0141
30	70	0.7	0.1	-0.3773	-0.3752	-0.3744	-0.3844	-0.0056	-0.0049
40	70	0.2	0.1	-0.3903	-0.3882	-0.3878	-0.3960	-0.0023	-0.0110
40	70	0.5	0.3	-0.3768	-0.3754	-0.3747	-0.3751	-0.0067	-0.0083
40	70	0.7	0.1	-0.3745	-0.3734	-0.3722	-0.3766	-0.0086	-0.0062

Exogenous variable β: Bias

T	N	α	ρ	SMLE	SDMLE	SDQMLE	LSDV	DIF-GMM	SYS-GMM
10	20	0.2	0.1	-0.0487	-0.0387	-0.0358	-0.0365	0.0098	-0.0062
10	20	0.5	0.3	-0.0329	-0.0258	-0.0219	-0.0103	0.0088	-0.0013
10	20	0.7	0.1	-0.0234	-0.0189	-0.0129	-0.0118	0.0318	0.0517
20	20	0.2	0.1	-0.0581	-0.0512	-0.0499	-0.0218	-0.0029	-0.0145
20	20	0.5	0.3	-0.0390	-0.0344	-0.0307	-0.0308	0.0017	-0.0053
20	20	0.7	0.1	-0.0316	-0.0260	-0.0222	-0.0073	0.0096	0.0163
30	20	0.2	0.1	-0.0572	-0.0522	-0.0514	0.0466	0.0001	-0.0098
30	20	0.5	0.3	-0.0403	-0.0362	-0.0340	-0.0434	0.0007	-0.0068
30	20	0.7	0.1	-0.0342	-0.0298	-0.0278	0.0025	0.0008	0.0091
40	20	0.2	0.1	-0.0553	-0.0513	-0.0507	-0.0527	-0.0017	-0.0032
40	20	0.5	0.3	-0.0381	-0.0355	-0.0338	0.0631	0.0017	-0.0017
40	20	0.7	0.1	-0.0330	-0.0298	-0.0278	0.0089	-0.0028	0.0052
10	30	0.2	0.1	-0.0572	-0.0457	-0.0437	-0.0560	-0.0046	-0.0185
10	30	0.5	0.3	-0.0311	-0.0225	-0.0200	0.0162	0.0072	0.0097
10	30	0.7	0.1	-0.0263	-0.0164	-0.0149	-0.0252	0.0142	0.0132
20	30	0.2	0.1	-0.0553	-0.0489	-0.0477	-0.0546	-0.0022	-0.0069
20	30	0.5	0.3	-0.0344	-0.0298	-0.0264	-0.0326	0.0014	0.0053
20	30	0.7	0.1	-0.0326	-0.0275	-0.0231	-0.0040	0.0062	0.0129
30	30	0.2	0.1	-0.0554	-0.0504	-0.0496	0.0309	-0.0006	-0.0074
30	30	0.5	0.3	-0.0373	-0.0338	-0.0319	-0.0284	0.0009	0.0034
30	30	0.7	0.1	-0.0340	-0.0297	-0.0274	0.0084	0.0010	0.0090
40	30	0.2	0.1	-0.0574	-0.0536	-0.0529	-0.0463	-0.0036	-0.0077
40	30	0.5	0.3	-0.0393	-0.0364	-0.0347	-0.0177	-0.0009	-0.0030
40	30	0.7	0.1	-0.0349	-0.0320	-0.0291	-0.0118	0.0002	0.0044
10	50	0.2	0.1	-0.0512	-0.0389	-0.0382	-0.0500	0.0017	-0.0049
10	50	0.5	0.3	-0.0353	-0.0255	-0.0238	-0.0043	0.0050	0.0131
10	50	0.7	0.1	-0.0262	-0.0174	-0.0156	-0.0204	0.0040	0.0148
20	50	0.2	0.1	-0.0557	-0.0486	-0.0476	-0.0523	-0.0018	-0.0075
20	50	0.5	0.3	-0.0384	-0.0330	-0.0303	-0.0381	-0.0004	-0.0027
20	50	0.7	0.1	-0.0323	-0.0256	-0.0232	-0.0309	0.0034	0.0130
30	50	0.2	0.1	-0.0556	-0.0507	-0.0498	-0.0287	-0.0003	-0.0038
30	50	0.5	0.3	-0.0390	-0.0355	-0.0332	-0.0293	-0.0004	0.0029
30	50	0.7	0.1	-0.0331	-0.0293	-0.0265	-0.0173	-0.0009	0.0082
40	50	0.2	0.1	-0.0558	-0.0520	-0.0513	-0.0372	0.0002	-0.0029
40	50	0.5	0.3	-0.0395	-0.0368	-0.0350	-0.0366	-0.0018	0.0019
40	50	0.7	0.1	-0.0334	-0.0307	-0.0280	0.0445	-0.0012	0.0069
10	70	0.2	0.1	-0.0512	-0.0391	-0.0386	-0.0457	0.0018	-0.0024
10	70	0.5	0.3	-0.0328	-0.0228	-0.0210	-0.0303	0.0027	0.0059
10	70	0.7	0.1	-0.0240	-0.0148	-0.0123	-0.0119	0.0056	0.0172
20	70	0.2	0.1	-0.0552	-0.0478	-0.0472	-0.0544	0.0001	-0.0059
20	70	0.5	0.3	-0.0374	-0.0301	-0.0294	-0.0335	-0.0015	0.0049
20	70	0.7	0.1	-0.0330	-0.0251	-0.0241	-0.0259	0.0022	0.0100
30	70	0.2	0.1	-0.0561	-0.0514	-0.0507	-0.0467	-0.0007	-0.0018
30	70	0.5	0.3	-0.0381	-0.0341	-0.0325	-0.0164	0.0006	0.0008
30	70	0.7	0.1	-0.0332	-0.0284	-0.0266	-0.0198	-0.0002	0.0065
40	70	0.2	0.1	-0.0567	-0.0528	-0.0522	-0.0400	-0.0013	-0.0048
40	70	0.5	0.3	-0.0395	-0.0368	-0.0349	-0.0363	-0.0025	0.0001
40	70	0.7	0.1	-0.0334	-0.0305	-0.0281	-0.0293	0.0003	0.0055

6.D Monte Carlo Results: RMSE

Time lag variable α: RMSE

T	N	α	ρ	SMLE	SDMLE	SDQMLE	LSDV	DIF-GMM	SYS-GMM
10	20	0.2	0.1	0.0135	0.0092	0.0085	0.0108	0.0235	0.0177
10	20	0.5	0.3	0.0074	0.0051	0.0038	0.0050	0.0207	0.0125
10	20	0.7	0.1	0.0062	0.0040	0.0026	0.0033	0.1781	0.0173
20	20	0.2	0.1	0.0089	0.0070	0.0067	0.0025	0.0048	0.0063
20	20	0.5	0.3	0.0039	0.0031	0.0026	0.0043	0.0059	0.0063
20	20	0.7	0.1	0.0026	0.0020	0.0016	0.0009	0.0085	0.0051
30	20	0.2	0.1	0.0078	0.0065	0.0063	0.0040	0.0026	0.0039
30	20	0.5	0.3	0.0031	0.0026	0.0023	0.0040	0.0027	0.0036
30	20	0.7	0.1	0.0018	0.0015	0.0013	0.0004	0.0032	0.0033
40	20	0.2	0.1	0.0071	0.0062	0.0061	0.0068	0.0020	0.0030
40	20	0.5	0.3	0.0027	0.0023	0.0022	0.0046	0.0020	0.0030
40	20	0.7	0.1	0.0015	0.0013	0.0012	0.0002	0.0016	0.0022
10	30	0.2	0.1	0.0126	0.0082	0.0077	0.0122	0.0094	0.0096
10	30	0.5	0.3	0.0069	0.0039	0.0033	0.0009	0.0183	0.0126
10	30	0.7	0.1	0.0057	0.0026	0.0021	0.0049	0.0282	0.0087
20	30	0.2	0.1	0.0088	0.0069	0.0066	0.0088	0.0033	0.0047
20	30	0.5	0.3	0.0037	0.0029	0.0024	0.0051	0.0032	0.0042
20	30	0.7	0.1	0.0024	0.0018	0.0014	0.0003	0.0056	0.0043
30	30	0.2	0.1	0.0075	0.0063	0.0061	0.0030	0.0017	0.0024
30	30	0.5	0.3	0.0030	0.0025	0.0022	0.0014	0.0018	0.0027
30	30	0.7	0.1	0.0017	0.0014	0.0012	0.0003	0.0018	0.0023
40	30	0.2	0.1	0.0070	0.0062	0.0060	0.0046	0.0012	0.0018
40	30	0.5	0.3	0.0025	0.0022	0.0020	0.0008	0.0011	0.0017
40	30	0.7	0.1	0.0015	0.0012	0.0012	0.0002	0.0014	0.0019
10	50	0.2	0.1	0.0121	0.0074	0.0071	0.0120	0.0046	0.0054
10	50	0.5	0.3	0.0066	0.0032	0.0031	0.0041	0.0119	0.0074
10	50	0.7	0.1	0.0053	0.0023	0.0019	0.0039	0.0119	0.0058
20	50	0.2	0.1	0.0083	0.0064	0.0062	0.0075	0.0018	0.0025
20	50	0.5	0.3	0.0035	0.0026	0.0023	0.0050	0.0020	0.0025
20	50	0.7	0.1	0.0023	0.0015	0.0013	0.0022	0.0028	0.0027
30	50	0.2	0.1	0.0074	0.0062	0.0060	0.0026	0.0011	0.0017
30	50	0.5	0.3	0.0029	0.0024	0.0021	0.0014	0.0011	0.0016
30	50	0.7	0.1	0.0017	0.0014	0.0011	0.0005	0.0011	0.0014
40	50	0.2	0.1	0.0069	0.0060	0.0059	0.0036	0.0007	0.0012
40	50	0.5	0.3	0.0026	0.0022	0.0020	0.0036	0.0007	0.0010
40	50	0.7	0.1	0.0014	0.0012	0.0011	0.0020	0.0007	0.0011
10	70	0.2	0.1	0.0116	0.0068	0.0067	0.0100	0.0031	0.0038
10	70	0.5	0.3	0.0064	0.0031	0.0029	0.0084	0.0060	0.0052
10	70	0.7	0.1	0.0051	0.0021	0.0017	0.0017	0.0094	0.0057
20	70	0.2	0.1	0.0084	0.0064	0.0062	0.0084	0.0011	0.0018
20	70	0.5	0.3	0.0035	0.0023	0.0023	0.0041	0.0014	0.0020
20	70	0.7	0.1	0.0023	0.0013	0.0012	0.0014	0.0019	0.0024
30	70	0.2	0.1	0.0073	0.0061	0.0059	0.0052	0.0006	0.0009
30	70	0.5	0.3	0.0027	0.0022	0.0020	0.0007	0.0007	0.0011
30	70	0.7	0.1	0.0016	0.0012	0.0011	0.0005	0.0007	0.0010
40	70	0.2	0.1	0.0069	0.0060	0.0059	0.0038	0.0005	0.0009
40	70	0.5	0.3	0.0025	0.0022	0.0020	0.0037	0.0005	0.0008
40	70	0.7	0.1	0.0014	0.0012	0.0010	0.0011	0.0005	0.0008

Spatial lag variable ρ: RMSEs

T	N	α	ρ	SMLE	SDMLE	SDQMLE	LSDV	DIF-GMM	SYS-GMM
10	20	0.2	0.1	0.0018	0.0018	0.0018	0.0078	0.0979	0.0324
10	20	0.5	0.3	0.0015	0.0014	0.0015	0.0093	0.0333	0.0153
10	20	0.7	0.1	0.0015	0.0015	0.0016	0.0078	0.2125	0.0129
20	20	0.2	0.1	0.0011	0.0011	0.0011	0.0041	0.0320	0.0219
20	20	0.5	0.3	0.0008	0.0007	0.0007	0.0031	0.0100	0.0058
20	20	0.7	0.1	0.0006	0.0006	0.0006	0.0052	0.0331	0.0043
30	20	0.2	0.1	0.0005	0.0005	0.0005	0.0231	0.0111	0.0105
30	20	0.5	0.3	0.0005	0.0005	0.0005	0.0008	0.0046	0.0039
30	20	0.7	0.1	0.0003	0.0003	0.0003	0.0061	0.0054	0.0038
40	20	0.2	0.1	0.0005	0.0005	0.0005	0.0012	0.0075	0.0080
40	20	0.5	0.3	0.0005	0.0004	0.0005	0.0010	0.0020	0.0020
40	20	0.7	0.1	0.0002	0.0002	0.0002	0.0004	0.0022	0.0014
10	30	0.2	0.1	0.0015	0.0015	0.0015	0.0026	0.0393	0.0304
10	30	0.5	0.3	0.0011	0.0010	0.0010	0.0045	0.0252	0.0099
10	30	0.7	0.1	0.0010	0.0010	0.0011	0.0012	0.0307	0.0090
20	30	0.2	0.1	0.0006	0.0006	0.0006	0.0016	0.0122	0.0101
20	30	0.5	0.3	0.0006	0.0005	0.0005	0.0030	0.0082	0.0046
20	30	0.7	0.1	0.0003	0.0003	0.0003	0.0007	0.0108	0.0027
30	30	0.2	0.1	0.0004	0.0004	0.0004	0.0103	0.0061	0.0050
30	30	0.5	0.3	0.0006	0.0005	0.0005	0.0004	0.0029	0.0018
30	30	0.7	0.1	0.0002	0.0002	0.0002	0.0004	0.0027	0.0016
40	30	0.2	0.1	0.0003	0.0003	0.0003	0.0005	0.0059	0.0049
40	30	0.5	0.3	0.0004	0.0004	0.0004	0.0006	0.0016	0.0012
40	30	0.7	0.1	0.0002	0.0002	0.0002	0.0002	0.0033	0.0016
10	50	0.2	0.1	0.0009	0.0007	0.0009	0.0021	0.0140	0.0101
10	50	0.5	0.3	0.0007	0.0005	0.0006	0.0143	0.0771	0.0083
10	50	0.7	0.1	0.0006	0.0005	0.0006	0.0014	0.0174	0.0046
20	50	0.2	0.1	0.0004	0.0003	0.0003	0.0008	0.0094	0.0057
20	50	0.5	0.3	0.0005	0.0004	0.0004	0.0025	0.0032	0.0025
20	50	0.7	0.1	0.0002	0.0002	0.0002	0.0007	0.0058	0.0016
30	50	0.2	0.1	0.0003	0.0003	0.0003	0.0035	0.0042	0.0044
30	50	0.5	0.3	0.0004	0.0003	0.0003	0.0002	0.0019	0.0016
30	50	0.7	0.1	0.0001	0.0001	0.0001	0.0002	0.0016	0.0011
40	50	0.2	0.1	0.0002	0.0002	0.0002	0.0024	0.0030	0.0029
40	50	0.5	0.3	0.0004	0.0004	0.0004	0.0029	0.0012	0.0010
40	50	0.7	0.1	0.0002	0.0001	0.0002	0.0006	0.0012	0.0006
10	70	0.2	0.1	0.0005	0.0004	0.0004	0.0016	0.0143	0.0142
10	70	0.5	0.3	0.0006	0.0004	0.0005	0.0032	0.0199	0.0048
10	70	0.7	0.1	0.0004	0.0003	0.0004	0.0008	0.0121	0.0025
20	70	0.2	0.1	0.0002	0.0002	0.0002	0.0008	0.0050	0.0050
20	70	0.5	0.3	0.0005	0.0003	0.0004	0.0024	0.0024	0.0015
20	70	0.7	0.1	0.0002	0.0002	0.0002	0.0002	0.0032	0.0018
30	70	0.2	0.1	0.0002	0.0002	0.0002	0.0004	0.0026	0.0026
30	70	0.5	0.3	0.0004	0.0003	0.0003	0.0004	0.0012	0.0010
30	70	0.7	0.1	0.0001	0.0001	0.0001	0.0003	0.0009	0.0006
40	70	0.2	0.1	0.0001	0.0001	0.0001	0.0010	0.0020	0.0020
40	70	0.5	0.3	0.0004	0.0003	0.0003	0.0033	0.0007	0.0005
40	70	0.7	0.1	0.0001	0.0001	0.0001	0.0002	0.0007	0.0005

Endogenous variable γ: RMSE

T	N	α	ρ	SMLE	SDMLE	SDQMLE	LSDV	DIF-GMM	SYS-GMM
10	20	0.2	0.1	0.1733	0.1686	0.1676	0.1723	0.1073	0.1195
10	20	0.5	0.3	0.1634	0.1611	0.1593	0.1649	0.0892	0.0810
10	20	0.7	0.1	0.1603	0.1581	0.1572	0.1650	0.1134	0.0668
20	20	0.2	0.1	0.1615	0.1584	0.1579	0.1735	0.0424	0.0476
20	20	0.5	0.3	0.1499	0.1481	0.1470	0.1512	0.0329	0.0364
20	20	0.7	0.1	0.1477	0.1457	0.1448	0.1518	0.0288	0.0292
30	20	0.2	0.1	0.1549	0.1527	0.1524	0.1843	0.0248	0.0305
30	20	0.5	0.3	0.1446	0.1432	0.1425	0.1484	0.0227	0.0244
30	20	0.7	0.1	0.1431	0.1403	0.1408	0.1603	0.0173	0.0188
40	20	0.2	0.1	0.1529	0.1512	0.1509	0.1528	0.0184	0.0231
40	20	0.5	0.3	0.1441	0.1430	0.1425	0.1825	0.0142	0.0177
40	20	0.7	0.1	0.1408	0.1383	0.1387	0.1508	0.0122	0.0139
10	30	0.2	0.1	0.1754	0.1702	0.1696	0.1748	0.0634	0.0655
10	30	0.5	0.3	0.1608	0.1561	0.1569	0.1947	0.0521	0.0497
10	30	0.7	0.1	0.1596	0.1496	0.1564	0.1618	0.0468	0.0420
20	30	0.2	0.1	0.1603	0.1572	0.1567	0.1593	0.0292	0.0345
20	30	0.5	0.3	0.1500	0.1483	0.1471	0.1487	0.0188	0.0219
20	30	0.7	0.1	0.1469	0.1451	0.1439	0.1577	0.0201	0.0211
30	30	0.2	0.1	0.1549	0.1528	0.1524	0.1962	0.0152	0.0192
30	30	0.5	0.3	0.1455	0.1440	0.1434	0.1611	0.0110	0.0137
30	30	0.7	0.1	0.1433	0.1420	0.1408	0.1569	0.0115	0.0123
40	30	0.2	0.1	0.1540	0.1524	0.1521	0.1592	0.0125	0.0149
40	30	0.5	0.3	0.1433	0.1423	0.1417	0.1551	0.0094	0.0115
40	30	0.7	0.1	0.1418	0.1404	0.1396	0.1488	0.0080	0.0089
10	50	0.2	0.1	0.1691	0.1633	0.1632	0.1678	0.0354	0.0372
10	50	0.5	0.3	0.1597	0.1499	0.1555	0.1568	0.0365	0.0358
10	50	0.7	0.1	0.1560	0.1525	0.1528	0.1587	0.0254	0.0239
20	50	0.2	0.1	0.1601	0.1569	0.1565	0.1610	0.0146	0.0183
20	50	0.5	0.3	0.1491	0.1469	0.1462	0.1478	0.0129	0.0146
20	50	0.7	0.1	0.1456	0.1387	0.1426	0.1457	0.0098	0.0105
30	50	0.2	0.1	0.1552	0.1530	0.1527	0.1616	0.0097	0.0130
30	50	0.5	0.3	0.1449	0.1437	0.1428	0.1592	0.0075	0.0090
30	50	0.7	0.1	0.1420	0.1408	0.1396	0.1475	0.0065	0.0069
40	50	0.2	0.1	0.1531	0.1514	0.1511	0.1572	0.0066	0.0087
40	50	0.5	0.3	0.1434	0.1424	0.1418	0.1429	0.0057	0.0067
40	50	0.7	0.1	0.1409	0.1399	0.1387	0.1597	0.0047	0.0052
10	70	0.2	0.1	0.1691	0.1623	0.1633	0.1708	0.0252	0.0281
10	70	0.5	0.3	0.1605	0.1548	0.1563	0.1576	0.0224	0.0227
10	70	0.7	0.1	0.1539	0.1510	0.1503	0.1650	0.0186	0.0190
20	70	0.2	0.1	0.1594	0.1562	0.1558	0.1589	0.0107	0.0140
20	70	0.5	0.3	0.1494	0.1420	0.1464	0.1512	0.0081	0.0098
20	70	0.7	0.1	0.1457	0.1378	0.1426	0.1489	0.0076	0.0086
30	70	0.2	0.1	0.1557	0.1535	0.1532	0.1592	0.0061	0.0076
30	70	0.5	0.3	0.1451	0.1437	0.1430	0.1601	0.0053	0.0067
30	70	0.7	0.1	0.1426	0.1400	0.1403	0.1484	0.0044	0.0047
40	70	0.2	0.1	0.1527	0.1510	0.1508	0.1570	0.0048	0.0063
40	70	0.5	0.3	0.1425	0.1415	0.1408	0.1410	0.0037	0.0045
40	70	0.7	0.1	0.1407	0.1397	0.1389	0.1421	0.0032	0.0035

Exogenous variable β: RMSE

T	N	α	ρ	SMLE	SDMLE	SDQMLE	LSDV	DIF-GMM	SYS-GMM
10	20	0.2	0.1	0.0061	0.0053	0.0051	0.0055	0.0131	0.0386
10	20	0.5	0.3	0.0051	0.0047	0.0044	0.0049	0.0119	0.0169
10	20	0.7	0.1	0.0046	0.0044	0.0042	0.0047	0.0355	0.0335
20	20	0.2	0.1	0.0049	0.0041	0.0040	0.0028	0.0050	0.0120
20	20	0.5	0.3	0.0031	0.0027	0.0025	0.0028	0.0050	0.0101
20	20	0.7	0.1	0.0024	0.0021	0.0019	0.0018	0.0053	0.0095
30	20	0.2	0.1	0.0044	0.0038	0.0037	0.0042	0.0034	0.0072
30	20	0.5	0.3	0.0026	0.0023	0.0022	0.0030	0.0032	0.0066
30	20	0.7	0.1	0.0021	0.0018	0.0017	0.0014	0.0027	0.0075
40	20	0.2	0.1	0.0039	0.0034	0.0034	0.0036	0.0024	0.0055
40	20	0.5	0.3	0.0023	0.0021	0.0020	0.0058	0.0024	0.0059
40	20	0.7	0.1	0.0018	0.0016	0.0015	0.0010	0.0024	0.0054
10	30	0.2	0.1	0.0056	0.0045	0.0043	0.0055	0.0080	0.0142
10	30	0.5	0.3	0.0037	0.0031	0.0031	0.0050	0.0082	0.0177
10	30	0.7	0.1	0.0029	0.0024	0.0025	0.0029	0.0088	0.0128
20	30	0.2	0.1	0.0042	0.0035	0.0034	0.0041	0.0033	0.0080
20	30	0.5	0.3	0.0022	0.0019	0.0017	0.0021	0.0034	0.0071
20	30	0.7	0.1	0.0022	0.0019	0.0017	0.0016	0.0033	0.0076
30	30	0.2	0.1	0.0039	0.0034	0.0033	0.0023	0.0021	0.0044
30	30	0.5	0.3	0.0021	0.0019	0.0017	0.0017	0.0019	0.0059
30	30	0.7	0.1	0.0018	0.0016	0.0014	0.0010	0.0020	0.0048
40	30	0.2	0.1	0.0038	0.0034	0.0033	0.0028	0.0016	0.0027
40	30	0.5	0.3	0.0020	0.0018	0.0016	0.0010	0.0015	0.0031
40	30	0.7	0.1	0.0017	0.0015	0.0014	0.0008	0.0014	0.0044
10	50	0.2	0.1	0.0043	0.0032	0.0032	0.0043	0.0042	0.0070
10	50	0.5	0.3	0.0027	0.0020	0.0020	0.0019	0.0043	0.0097
10	50	0.7	0.1	0.0021	0.0017	0.0017	0.0021	0.0047	0.0088
20	50	0.2	0.1	0.0037	0.0030	0.0029	0.0033	0.0020	0.0051
20	50	0.5	0.3	0.0020	0.0016	0.0015	0.0020	0.0018	0.0045
20	50	0.7	0.1	0.0017	0.0013	0.0012	0.0016	0.0019	0.0050
30	50	0.2	0.1	0.0036	0.0030	0.0030	0.0014	0.0012	0.0029
30	50	0.5	0.3	0.0019	0.0016	0.0015	0.0014	0.0012	0.0032
30	50	0.7	0.1	0.0015	0.0013	0.0011	0.0008	0.0011	0.0030
40	50	0.2	0.1	0.0035	0.0030	0.0030	0.0018	0.0009	0.0023
40	50	0.5	0.3	0.0019	0.0017	0.0016	0.0017	0.0008	0.0020
40	50	0.7	0.1	0.0014	0.0012	0.0011	0.0025	0.0008	0.0027
10	70	0.2	0.1	0.0037	0.0026	0.0025	0.0032	0.0030	0.0050
10	70	0.5	0.3	0.0021	0.0016	0.0015	0.0020	0.0029	0.0071
10	70	0.7	0.1	0.0016	0.0013	0.0012	0.0014	0.0036	0.0075
20	70	0.2	0.1	0.0036	0.0029	0.0028	0.0036	0.0014	0.0035
20	70	0.5	0.3	0.0019	0.0013	0.0013	0.0016	0.0012	0.0037
20	70	0.7	0.1	0.0014	0.0010	0.0010	0.0011	0.0012	0.0041
30	70	0.2	0.1	0.0034	0.0029	0.0028	0.0025	0.0009	0.0018
30	70	0.5	0.3	0.0018	0.0015	0.0013	0.0007	0.0009	0.0022
30	70	0.7	0.1	0.0014	0.0011	0.0010	0.0007	0.0008	0.0021
40	70	0.2	0.1	0.0035	0.0030	0.0030	0.0019	0.0006	0.0017
40	70	0.5	0.3	0.0017	0.0015	0.0014	0.0015	0.0006	0.0015
40	70	0.7	0.1	0.0013	0.0012	0.0010	0.0011	0.0006	0.0020